Sums of Series Exam Questions (Edexcel)

Q1.

(a) Show, using the formulae for $\sum r$ and $\sum r^2$, that

$$\sum_{r=1}^{n} (6r^2 + 4r - 1) = n(n + 2)(2n + 1)$$

(5)

(b) Hence, or otherwise, find the value of $\sum_{r=11}^{20} (6r^2 + 4r - 1)$.

(2)

(Total 7 marks)
(Q02 6667/01, Jan 2009)

Q2.

(a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n} (5r-2)^2 = \frac{1}{6}n(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of k for which

$$\sum_{r=1}^{k} (5r-2)^2 = 94k^2$$

(4)

(Total for question = 9 marks)
(Q03 8FM0/01, Oct 2021)

Q3.

(a) Using the formula for $\sum_{r=1}^{n} r^2$ write down, in terms of n only, an expression for

$$\sum_{r=1}^{3n} r^2$$

(1)

(b) Show that, for all integers n, where $n > 0$

$$\sum_{r=2n+1}^{3n} r^2 = \frac{n}{6}(an^2 + bn + c)$$

where the values of the constants a, b and c are to be found.

(4)

(Total for question = 5 marks)
(Q03 6667/01, June 2016)

Q4.

(a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{n}{3}(an^2 - 1)$$

where a is a constant to be determined.

(5)

(b) Hence determine the sum of the squares of all positive odd three-digit integers.

(3)

(Total for question = 8 marks)

(Q08 8FM0/01, June 2023)

Q5.

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (3r^2 + 8r + 3) = \frac{1}{2}n(2n+5)(n+3)$$

for all positive integers n.

(5)

Given that

$$\sum_{r=1}^{12} \left(3r^2 + 8r + 3 + k(2^{r-1})\right) = 3520$$

(b) find the exact value of the constant k.

(4)

(Total for question = 9 marks)

(Q07 6667/01, June 2017)

Q6.

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \tfrac{1}{3}n(4n^2-1)$$

(6)

(b) Hence show that

$$\sum_{r=2n+1}^{4n} (2r-1)^2 = an(bn^2-1)$$

where a and b are constants to be found.

(3)

(Total 9 marks)

(Q05 6667/01, June 2014)

Q7.

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} (r+2)(r+3) = \frac{1}{3}n(n^2+9n+26)$$

for all positive integers n.

(6)

(b) Hence show that

$$\sum_{r=n+1}^{3n} (r+2)(r+3) = \frac{2}{3}n(an^2+bn+c)$$

where a, b and c are integers to be found.

(4)

(Total 10 marks)

(Q04 6667/01, June 2013)

Q8.

(a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(3r-2)^2 = \frac{1}{2}n\left[6n^2-3n-1\right]$$

for all positive integers n.

(5)

(b) Hence find any values of n for which

$$\sum_{r=5}^{n}(3r-2)^2 + 103\sum_{r=1}^{28} r\cos\left(\frac{r\pi}{2}\right) = 3n^3$$

(5)

(Total for question = 10 marks)

(Q06 8FM0/01, June 2018)

Q9.

(a) Use the standard summation formulae to show that, for $n \in \mathbb{N}$,

$$\sum_{r=1}^{n}\left(3r^2-17r-25\right) = n\left(n^2-An-B\right)$$

where A and B are integers to be determined.

(4)

(b) Explain why, for $k \in \mathbb{N}$,

$$\sum_{r=1}^{3k} r\tan(60r)^\circ = -k\sqrt{3}$$

(2)

Using the results from part (a) and part (b) and showing all your working,

(c) determine any value of n that satisfies

$$\sum_{r=5}^{n}\left(3r^2-17r-25\right) = 15\left[\sum_{r=6}^{3n} r\tan(60r)^\circ\right]^2$$

(6)

(Total for question = 12 marks)

(Q05 8FM0/01, June 2022)

Q10.

An art display consists of an arrangement of n marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams.
The mass of each subsequent marble is 3 grams more than the mass of the previous one, so that the rth marble has mass $(7 + 3r)$ grams.

(a) Show that the mean mass, in grams, of the marbles in the display is given by

$$\frac{1}{2}(3n+17)$$

(3)

Given that there are 85 marbles in the display,

(b) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

(6)

(Total for question = 9 marks)

(Q06 8FM0/01, June 2019)