Matrix Operations, Determinants and Inverses (From OCR 4755)

Q1, (Jan 2006, Q1)

You are given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Calculate, where possible, $2\mathbf{B}$, $\mathbf{A}+\mathbf{C}$, $\mathbf{CA}$ and $\mathbf{A}-\mathbf{B}$. [5]

(ii) Show that matrix multiplication is not commutative. [2]

Q2, (Jun 2007, Q1i)

You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(i) Find the inverse of $\mathbf{M}$. [2]

Q3, (Jan 2007, Q9)

Matrices $\mathbf{M}$ and $\mathbf{N}$ are given by $\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$ and $\mathbf{N}^{-1}$. [3]

(ii) Find $\mathbf{MN}$ and $(\mathbf{MN})^{-1}$. Verify that $(\mathbf{MN})^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1}$. [6]

(iii) The result $(\mathbf{PQ})^{-1} = \mathbf{Q}^{-1}\mathbf{P}^{-1}$ is true for any two 2×2, non-singular matrices $\mathbf{P}$ and $\mathbf{Q}$.

The first two lines of a proof of this general result are given below. Beginning with these two lines, complete the general proof.

$$(\mathbf{PQ})^{-1}\mathbf{PQ} = \mathbf{I}$$

$$\Rightarrow \quad (\mathbf{PQ})^{-1}\mathbf{PQQ}^{-1} = \mathbf{IQ}^{-1}$$ [4]

Q4, (Jun 2009, Q1)

(i) Find the inverse of the matrix $\mathbf{M} = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}$. **[2]**

(ii) Use this inverse to solve the simultaneous equations

$$4x - y = 49,$$
$$3x + 2y = 100,$$

showing your working clearly. **[3]**

Q5, (Jun 2015, Q1)

Given that $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, where $\mathbf{M} = \begin{pmatrix} 4 & -3 \\ 8 & 21 \end{pmatrix}$, find x and y. **[6]**

Q6, (Jun 2010, Q2)

You are given that $\mathbf{M} = \begin{pmatrix} 2 & -5 \\ 3 & 7 \end{pmatrix}$.

$\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -1 \end{pmatrix}$ represents two simultaneous equations.

(i) Write down these two equations. **[2]**

(ii) Find $\mathbf{M}^{-1}$ and use it to solve the equations. **[4]**

Q7, (Jun 2011, Q9)

The simultaneous equations

$$2x - y = 1$$
$$3x + ky = b$$

are represented by the matrix equation $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ b \end{pmatrix}$.

(i) Write down the matrix **M**. **[2]**

(ii) State the value of k for which $\mathbf{M}^{-1}$ does not exist and find $\mathbf{M}^{-1}$ in terms of k when $\mathbf{M}^{-1}$ exists.

Use $\mathbf{M}^{-1}$ to solve the simultaneous equations when $k = 5$ and $b = 21$. **[7]**

(iii) What can you say about the solutions of the equations when $k = -\frac{3}{2}$? **[1]**

(iv) The two equations can be interpreted as representing two lines in the x-y plane. Describe the relationship between these two lines

(A) when $k = 5$ and $b = 21$,

(B) when $k = -\frac{3}{2}$ and $b = 1$,

(C) when $k = -\frac{3}{2}$ and $b = \frac{3}{2}$. **[3]**