Matrices – 3x3 Transformations Exam Questions (Edexcel)

Q1.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometric transformation A represented by the matrix **A**.

(2)

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

The transformation B is represented by the matrix **B**.

The transformation A followed by the transformation B is the transformation C, which is represented by the matrix **C**.

To determine matrix **C**, a student attempts the following matrix multiplication.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

(b) State the error made by the student.

(1)

(c) Determine the correct matrix **C**.

(1)

(Total for question = 4 marks)

(Q03 8FM0/01, June 2023)

Q2.

*With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the y-axis is represented by the matrix*

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

The point P has coordinates (8, 3, 2)

The point Q is the image of P under the transformation reflection in the plane $y = 0$

(a) Write down the coordinates of Q

(1)

The point R is the image of P under the transformation rotation through 120° anticlockwise about the y-axis, with respect to the **right-hand rule**.

(b) Determine the exact coordinates of R

(2)

(c) Hence find $|\overrightarrow{PR}|$ giving your answer as a simplified surd.

(2)

(d) Show that $\overrightarrow{PR}$ and $\overrightarrow{PQ}$ are perpendicular.

(1)

(e) Hence determine the exact area of triangle PQR, giving your answer as a surd in simplest form.

(2)

(Total for question = 8 marks)

(Q03 8FM0/01, June 2022)

Q3.

With respect to the **right-hand rule**, a rotation through $\theta°$ anticlockwise about the z-axis is represented by the matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Given that the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a rotation through $\alpha°$ anticlockwise about the z-axis with respect to the **right-hand rule**,

(a) determine the value of α.

(1)

(b) Hence determine the smallest possible positive integer value of k for which $\mathbf{M}^k = \mathbf{I}$

(2)

The 3×3 matrix **N** represents a reflection in the plane with equation $y = 0$

(c) Write down the matrix **N**.

(1)

The point A has coordinates $(-2, 4, 3)$

The point B is the image of the point A under the transformation represented by matrix **M** followed by the transformation represented by matrix **N**.

(d) Show that the coordinates of B are $(2 + \sqrt{3}, 2\sqrt{3} - 1, 3)$

(2)

Given that O is the origin,

(e) show that, to 3 significant figures, the size of angle AOB is 66.9°

(2)

(f) Hence determine the area of triangle AOB, giving your answer to 3 significant figures.

(2)

(Total for question = 10 marks)

(Q02 8FM0/01, June 2024)