Matrices – 2x2 Transformations Exam Questions (Edexcel)

Q1.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

(2)

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

(3)

(Total for question = 5 marks)
(Q03 6667/01, June 2017)

Q2.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**.

(2)

The transformation U maps the point A, with coordinates (p, q), onto the point B, with coordinates $(6\sqrt{2}, 3\sqrt{2})$.

(b) Find the value of p and the value of q.

(3)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line with equation $y = x$.

(c) Write down the matrix **Q**.

(1)

The transformation U followed by the transformation V is the transformation T. The transformation T is represented by the matrix **R**.

(d) Find the matrix **R**.

(3)

(e) Deduce that the transformation T is self-inverse.

(1)

(Total for question = 10 marks)
(Q05 6667/01, June 2016)

Q3.

(i) $$\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}$$, where k is a real constant.

Given that **A** is a singular matrix, find the possible values of k.

(4)

(ii) $$\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$$

A triangle T is transformed onto a triangle T' by the transformation represented by the matrix **B**.

The vertices of triangle T' have coordinates (0, 0), (–20, 6) and ($10c$, $6c$), where c is a positive constant.

The area of triangle T' is 135 square units.

(a) Find the matrix $\mathbf{B}^{-1}$

(2)

(b) Find the coordinates of the vertices of the triangle T, in terms of c where necessary.

(3)

(c) Find the value of c.

(3)

(Total for question = 12 marks)
(Q06 6667/01, June 2015)

Q4.

(i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents an enlargement, scale factor −2, with centre the origin.

(b) Write down the matrix **B**.

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}$$, where k is a positive constant.

Triangle T has an area of 16 square units.

Triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of the triangle T' is 224 square units, find the value of k.

(3)

(Total 6 marks)

(Q04 6667/01/R, June 2014)

Q5.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(a) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(b) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(c) Find the matrix **Q**.

(2)

(Total 7 marks)

(Q05 6667/01/R, June 2013)

Q6.

The transformation U, represented by the 2 × 2 matrix **P**, is a rotation through 90º anticlockwise about the origin.

(a) Write down the matrix **P**.

(1)

The transformation V, represented by the 2 × 2 matrix **Q**, is a reflection in the line $y = -x$.

(b) Write down the matrix **Q**.

(1)

Given that U followed by V is transformation T, which is represented by the matrix **R**,

(c) express **R** in terms of **P** and **Q**,

(1)

(d) find the matrix **R**,

(2)

(e) give a full geometrical description of T as a single transformation.

(2)

(Total 7 marks)

(Q04 6667/01, Jan 2013)

Q7.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}$$

(a) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(b) Find the value of a.

(3)

The point R has coordinates (6, 0).
Given that O is the origin,

(c) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(d) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(f) Find **B**.

(4)

(Total 14 marks)
(Q06 6667/01, June 2012)

Q8.

A right angled triangle T has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When T is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is T'.

(a) Find the coordinates of the vertices of T'.

(2)

(b) Describe fully the transformation represented by **P**.

(2)

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When T is transformed by the matrix **QR**, the image is T''.

(c) Find **QR**.

(2)

(d) Find the determinant of **QR**.

(2)

(e) Using your answer to part (d), find the area of T''.

(3)

(Total 11 marks)

(Q03 6667/01, Jan 2012)

Q9.

(a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$$

(i) find $\mathbf{A}^2$,
(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

describe fully the geometrical transformation represented by **B**.

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

where k is a constant, find the value of k for which the matrix **C** is singular.

(3)

(Total 9 marks)
(Q03 6667/01, June 2011)

Q10.

(i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where p is a constant.

(a) Find, in terms of p, the matrix **AB**

(2)

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where k is a constant and **I** is the 2×2 identity matrix,

(b) find the value of p and the value of k.

(4)

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle T has an area of 15 square units.

Triangle T is transformed to the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 270 square units, find the possible values of a.

(5)

(Total for question = 11 marks)

(Q05 6667/01, June 2017)

Q11.

The transformation P is an enlargement, centre the origin, with scale factor k, where $k > 0$

The transformation Q is a rotation through angle θ degrees anticlockwise about the origin.

The transformation P followed by the transformation Q is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$$

(a) Determine

(i) the value of k,

(ii) the smallest value of θ

(4)

A square S has vertices at the points with coordinates (0, 0), (a, $-a$), ($2a$, 0) and (a, a) where a is a constant.

The square S is transformed to the square S' by the transformation represented by **M**.

(b) Determine, in terms of a, the area of S'

(2)

(Total for question = 6 marks)

(Q01 9FM0/01, Oct 2021)