Y1 Complex Numbers – Argand Diagram and Loci Exam Questions (Edexcel)

Q1.

The point P represents a complex number z on an Argand diagram such that

$$|z-6\text{i}| = 2|z-3|$$

(a) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z-6) = -\frac{3\pi}{4}$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(c) Find the complex number for which both

$$|z-6\text{i}| = 2|z-3| \text{ and } \arg(z-6) = -\frac{3\pi}{4}$$

(4)

(Total 14 marks)

(Q12 6668/01, June 2012)

Q2.

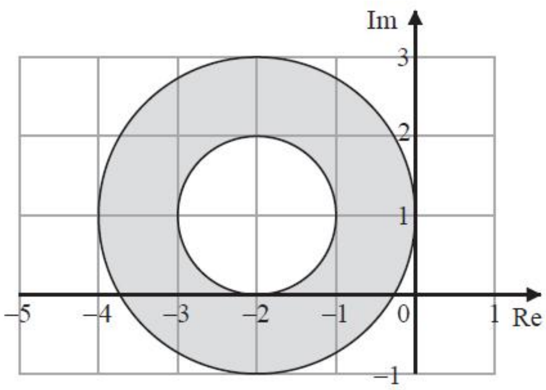

Figure 1

Figure 1 shows an Argand diagram.

The set P, of points that lie within the shaded region including its boundaries, is defined by

$$P = \{z \in \mathbb{C} : a \leqslant |z + b + c\text{i}| \leqslant d\}$$

where a, b, c and d are integers.

(a) Write down the values of a, b, c and d.

(3)

The set Q is defined by

$$Q = \{z \in \mathbb{C} : a \leqslant |z + b + c\text{i}| \leqslant d\} \cap \{z \in \mathbb{C} : |z - \text{i}| \leqslant |z - 3\text{i}|\}$$

(b) Determine the exact area of the region defined by Q, giving your answer in simplest form.

(7)

(Total for question = 10 marks)

(Q05 8FM0/01, Oct 2021)

Q3.

(a) Express the complex number $w = 4\sqrt{3} - 4\text{i}$ in the form $r(\cos\theta\ \theta + \text{i}\sin\theta)$ where $r > 0$ and $-\pi < \theta \leq \pi$

(4)

(b) Show, on a single Argand diagram,

(i) the point representing w

(ii) the locus of points defined by $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

(c) Hence determine the minimum distance of w from the locus $\arg(z + 10\text{i}) = \frac{\pi}{3}$

(3)

(Total for question = 10 marks)

(Q02 8FM0/01, June 2022)

Q4.

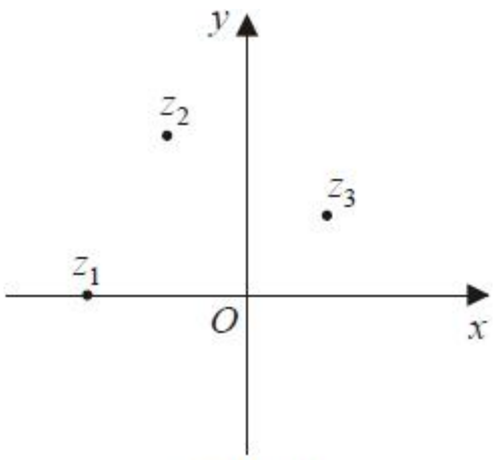

Figure 1

The complex numbers $z_1 = -\mathbf{2}$, $z_2 = -1 + \mathbf{2i}$ and $z_3 = 1 + \text{i}$ are plotted in Figure 1, on an Argand diagram for the complex plane with $z = x + \mathbf{i}y$

(a) Explain why z_1, z_2 and z_3,cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

(b) Show that $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \frac{\pi}{4}$

(3)

(c) Hence show that $\arctan 2 - \arctan\frac{1}{3} = \frac{\pi}{4}$

(2)

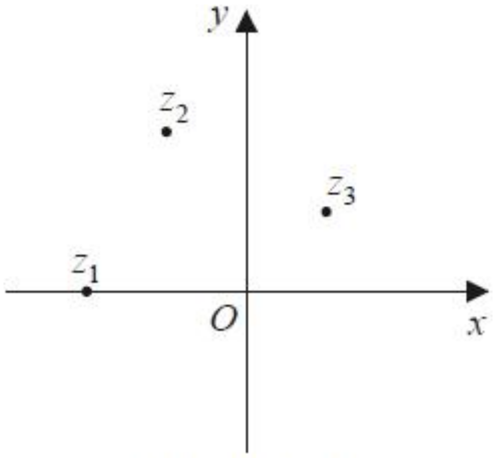

Diagram 1

(d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z+2| \leq |z-1-\mathrm{i}|$$

(2)

(Total for question = 9 marks)

(Q05 8FM0/01, June 2019)

Q5.

(a) (i) Show on an Argand diagram the locus of points given by the values of z satisfying

$$|z-4-3\mathrm{i}| = 5$$

Taking the initial line as the positive real axis with the pole at the origin and given that $\theta \in [\alpha, \alpha + \pi]$, where $\alpha = -\arctan\left(\frac{4}{3}\right)$,

(ii) show that this locus of points can be represented by the polar curve with equation

$$r = 8\cos\theta + 6\sin\theta$$

(6)

The set of points A is defined by

$$A = \left\{z : 0 \leq \arg z \leq \frac{\pi}{3}\right\} \cap \{z : |z-4-3\mathrm{i}| \leq 5\}$$

(b) (i) Show, by shading on your Argand diagram, the set of points A.

(ii) Find the **exact** area of the region defined by A, giving your answer in simplest form.

(7)

(Total for question = 13 marks)

(Q06 9FM0/02, Specimen papers)

Q6.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$z_1 = -4 + 4i$$

(a) Express z_1 in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r \in \mathbb{R}$, $r > 0$ and $0 \leq \theta < 2\pi$

(2)

$$z_2 = 3\left(\cos\frac{17\pi}{12} + \text{i}\sin\frac{17\pi}{12}\right)$$

(b) Determine in the form $a + \text{i}b$, where a and b are exact real numbers,

(i) $\dfrac{z_1}{z_2}$

(2)

(ii) $(z_2)^4$

(2)

(c) Show on a single Argand diagram

(i) the complex numbers z_1, z_2 and $\dfrac{z_1}{z_2}$

(ii) the region defined by $\{z \in \mathbb{C} : |z - z_1| < |z - z_2|\}$

(4)

(Total for question = 10 marks)

(Q03 9FM0/01, June 2023)