Wilcoxon Single Sample and Paired Sample Signed Rank Tests (From OCR 4768)

Q1, (Jun 2006, Q4i)

A company has many factories. It is concerned about incidents of trespassing and, in the hope of reducing if not eliminating these, has embarked on a programme of installing new fencing.

(i) Records for a random sample of 9 factories of the numbers of trespass incidents in typical weeks before and after installation of the new fencing are as follows.

Factory	A	B	C	D	E	F	G	H	I
Number before installation	8	12	6	4	14	22	4	13	14
Number after installation	6	11	0	1	18	10	11	5	4

Use a Wilcoxon test to examine at the 5% level of significance whether it appears that, on the whole, the number of trespass incidents per week is lower after the installation of the new fencing than before. [9]

Q2, (Jan 2007, Q4b)

An amateur weather forecaster has been keeping records of air pressure, measured in atmospheres.

The forecaster buys a new digital barometer that can be linked to her computer for easier recording of observations. She decides that she wishes to compare the readings of the new barometer with those of the old one. For a random sample of 10 days, the readings (in atmospheres) of the two barometers are shown below.

Day	A	B	C	D	E	F	G	H	I	J
Old	0.992	1.005	1.001	1.011	1.026	0.980	1.020	1.025	1.042	1.009
New	0.985	1.003	1.002	1.014	1.022	0.988	1.030	1.016	1.047	1.025

Use an appropriate Wilcoxon test to examine at the 10% level of significance whether there is any reason to suppose that, on the whole, readings on the old and new barometers do not agree. [9]

Q3, (Jun 2007, Q4ii) [Modified]

You are given that the median length between successive flaws in a roll of material is 124 metres. A sample of material is obtained and ten lengths (in metres) between flaws is noted.

239 77 179 221 100 312 52 129 236 42

Test at the 10% level of significance whether the median length may still be assumed to be 124 metres.

[9]

Q4, (Jan 2008, Q1b) [Modified]

For a certain task, the times (in milliseconds) taken by my computer on 10 randomly chosen occasions were as follows.

6.4 5.9 5.0 6.2 6.8 6.0 5.2 6.5 5.7 5.3

From past experience it is thought that the median time for this task is 5.4 milliseconds. Carry out a suitable Wilcoxon test at the 5% level of significance to investigate this, stating your hypotheses carefully.

[10]

Q5, (Jun 2008, Q3b)

A tea grower is testing two types of plant for the weight of tea they produce.

The tea grower deals with many types of tea and employs tasters to rate them. The tasters do this by giving each tea a score out of 100. The tea grower wishes to compare the scores given by two of the tasters. Their scores for a random selection of 10 teas are as follows.

Tea	Q	R	S	T	U	V	W	X	Y	Z
Taster 1	69	79	85	63	81	65	85	86	89	77
Taster 2	74	75	99	66	75	64	96	94	96	86

Use a Wilcoxon test to examine, at the 5% level of significance, whether it appears that, on the whole, the scores given to teas by these two tasters differ. [8]

Q6, (Jan 2009, Q1b)

Every day, Godfrey does a puzzle from the newspaper and records the time taken in minutes. Last year, his median time was 32 minutes. His times for a random sample of 12 puzzles this year are as follows.

40 20 18 11 47 36 38 35 22 14 12 21

Use an appropriate test, with a 5% significance level, to examine whether Godfrey's times this year have decreased on the whole. **[8]**

Q7, (Jun 2009, Q3)

A company which employs 600 staff wishes to improve its image by introducing new uniforms for the staff to wear. The human resources manager would like to obtain the views of the staff. She decides to do this by means of a systematic sample of 10% of the staff.

(i) How should she go about obtaining such a sample, ensuring that all members of staff are equally likely to be selected? Explain whether this constitutes a simple random sample. **[5]**

At a later stage in the process, the choice of uniform has been reduced to two possibilities. Twelve members of staff are selected to take part in deciding which of the two uniforms to adopt. Each of the twelve assesses each uniform for comfort, appearance and practicality, giving it a total score out of 10. The scores are as follows.

Staff member	1	2	3	4	5	6	7	8	9	10	11	12
Uniform A	4.2	2.6	10.0	9.0	8.2	2.8	5.0	7.4	2.8	6.8	10.0	9.8
Uniform B	5.0	5.2	1.4	2.8	2.2	6.4	7.4	7.8	6.8	1.2	3.4	7.6

A Wilcoxon signed rank test is to be used to decide whether there is any evidence of a preference for one of the uniforms.

(ii) Explain why this test is appropriate in these circumstances and state the hypotheses that should be used. **[4]**

(iii) Carry out the test at the 5% significance level. **[8]**

Q8, (Jun 2010, Q3a)

In order to prevent and/or control the spread of infectious diseases, the Government has various vaccination programmes. One such programme requires people to receive a booster injection at the age of 18. It is felt that the proportion of people receiving this booster could be increased and a publicity campaign is undertaken for this purpose. In order to assess the effectiveness of this campaign, health authorities across the country are asked to report the percentage of 18-year-olds receiving the booster before and after the campaign. The results for a randomly chosen sample of 9 authorities are as follows.

Authority	A	B	C	D	E	F	G	H	I
Before	76	98	88	81	86	84	83	93	80
After	82	97	93	77	83	95	91	95	89

This sample is to be tested to see whether the campaign appears to have been successful in raising the percentage receiving the booster.

(i) Explain why the use of paired data is appropriate in this context. **[1]**

(ii) Carry out an appropriate Wilcoxon signed rank test using these data, at the 5% significance level. **[10]**

Q9, (OCR 4735, Jun 2011, Q2)

A botanist believes that some species of plants produce more flowers at high altitudes than at low altitudes. In order to investigate this belief the botanist randomly samples 11 species of plants each of which occurs at both altitudes. The numbers of flowers on the plants are shown in the table.

Species	1	2	3	4	5	6	7	8	9	10	11
Number of flowers at low altitude	5	3	4	7	2	9	6	5	4	11	2
Number of flowers at high altitude	1	6	10	8	14	16	20	21	15	2	12

(i) Use the Wilcoxon signed rank test at the 5% significance level to test the botanist's belief. **[7]**

(ii) Explain why the Wilcoxon rank sum test should not be used for this test. **[1]**

Q10, (OCR 4735, Jun 2014, Q1)

A teacher believes that the calculator paper in a GCSE Mathematics examination was easier than the non-calculator paper. The marks of a random sample of ten students are shown in the table.

Student	A	B	C	D	E	F	G	H	I	J
Mark on paper 1 (non-calculator)	66	79	58	87	67	55	75	62	50	84
Mark on paper 2 (calculator)	57	84	70	90	75	42	82	72	65	82

(i) Use a Wilcoxon signed-rank test, at the 5% significance level, to test the teacher's belief. **[7]**

(ii) State the assumption necessary for this test to be applied. **[1]**

Q11, (Jan 2012, Q3b)

A further piece of research compares the incidence of myocardial infarction in men aged 55 to 70 with that in women aged 55 to 70. Incidence is measured by the number of infarctions per 10000 of the population. For a random sample of 8 health authorities across the UK, the following results for the year 2010 were obtained.

Health authority	A	B	C	D	E	F	G	H
Incidence in men	47	56	15	51	45	54	50	32
Incidence in women	36	30	30	47	54	55	27	27

A Wilcoxon paired sample test, using the hypotheses H_0: $m = 0$ and H_1: $m \neq 0$ where m is the population median difference, is to be carried out to investigate whether there is any difference between men and women on the whole.

(i) Explain why a paired test is being used in this context. **[1]**

(ii) Carry out the test using a 10% level of significance. **[8]**

Q12, (Jun 2015, Q1b)

At a large secondary school, the median number of half days absent per pupil per year (based on several years' records) was known to be 23. Last year the school carried out a drive to lower the number of absences. A random sample of 12 pupils had been absent for the following numbers of half days during the year.

14 10 15 13 35 9 24 19 30 26 29 8

A Wilcoxon single sample test is to be carried out to see if the drive has been successful.

(i) Why might a Wilcoxon test be appropriate? **[1]**

(ii) What distributional assumption is needed for the test? **[1]**

(iii) Carry out the test, using a 5% level of significance. **[10]**

Q13, (OCR 4735, Jun 2015, Q2)

The manufacturer of a painkiller, designed to relieve headaches, claims that people taking the painkiller feel relief in at most 30 minutes, on average. A random sample of eight users of the painkiller recorded the times it took for them to feel relief from their headaches. These times, in minutes, were as follows:

33 39 29 35 40 32 26 37

Use a Wilcoxon single-sample signed-rank test at the 5% significance level to test the manufacturer's claim, stating a necessary assumption. **[8]**

Q14, (Jun 2016, Q2b)

The median length of European fruit flies is 2.5 mm. South American fruit flies are believed to be larger than European fruit flies. A random sample of 12 South American fruit flies is taken. The flies are found to have the following lengths (in mm).

1.7 1.4 3.1 3.5 3.8 4.2 2.2 2.9 4.4 2.6 3.9 3.2

Carry out a Wilcoxon signed rank test, using a 5% level of significance, to test this belief. **[9]**

Q15, (Jun 2010, Q5)

In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries x, in thousands of pounds, of 50 randomly chosen employees were obtained.

(i) The values $|x - 19.5|$ were calculated and ranked. No two values of x were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of $(x - 19.5)$ was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. **[10]**

(ii) If the assumption you stated in part **(i)** does not hold, what test could have been used? **[1]**