Wilcoxon Rank-Sum Tests (From OCR 4735)

Q1, (Jun 2008, Q4)

William takes a bus regularly on the same journey, sometimes in the morning and sometimes in the afternoon. He wishes to compare morning and afternoon journey times. He records the journey times on 7 randomly chosen mornings and 8 randomly chosen afternoons. The results, each correct to the nearest minute, are as follows, where M denotes a morning time and A denotes an afternoon time.

M	A	A	M	M	M	M	M	M	A	A	A	A	A	A
19	20	22	24	25	26	28	30	31	33	35	37	38	39	42

William wishes to test for a difference between the average times of morning and afternoon journeys.

(ii) William chooses a non-parametric test at the 5% significance level. Carry out the test, stating the rejection region. [6]

Q2, (Jun 2011, Q5)

A test was carried out to compare the breaking strengths of two brands of elastic band, *A* and *B*, of the same size. Random samples of 6 were selected from each brand and the breaking strengths were measured. The results, in suitable units and arranged in ascending order for each brand, are as follows.

Brand *A*:	5.6	8.7	9.2	10.7	11.2	12.6
Brand *B*:	10.1	11.6	12.0	12.2	12.9	13.5

(i) Give one advantage that a non-parametric test might have over a parametric test in this context. **[1]**

(ii) Carry out a suitable Wilcoxon test at the 5% significance level of whether there is a difference between the average breaking strengths of the two brands. **[7]**

(iii) An extra elastic band of brand *B* was tested and found to have a breaking strength exceeding all of the other 12 bands. Determine whether this information alters the conclusion of your test. **[3]**

Q3, (Jun 2014, Q6)

A Wilcoxon rank-sum test with samples of sizes 11 and 12 is carried out.

(i) What is the least possible value of the test statistic W? **[2]**

(ii) The null hypothesis is that the two samples came from identical populations. Given that the null hypothesis was rejected at the 1% level using a 2-tail test, find the set of possible values of W. **[6]**

Q4, (Jun 2012, Q3i)

Because of the large number of students enrolled for a university geography course and the limited accommodation in the lecture theatre, the department provides a filmed lecture. Students are randomly assigned to two groups, one to attend the lecture theatre and the other the film. At the end of term the two groups are given the same examination. The geography professor wishes to test whether there is a difference in the performance of the two groups and selects the marks of two random samples of students, 6 from the group attending the lecture theatre and 7 from the group attending the films. The marks for the two samples, ordered for convenience, are shown below.

Lecture theatre:	30	36	48	51	59	62	
Filmed lecture:	40	49	52	56	63	64	68

(i) Stating a necessary assumption, carry out a suitable non-parametric test, at the 10% significance level, for a difference between the median marks of the two groups. **[7]**

Q5, (Jun 2013, Q4)

The effect of water salinity on the growth of a type of grass was studied by a biologist. A random sample of 22 seedlings was divided into two groups *A* and *B*, each of size 11.
Group *A* was treated with water of 0% salinity and group *B* was treated with water of 0.5% salinity. After three weeks the height (in cm) of each seedling was measured with the following results, which are ordered for convenience.

Group *A*	8.6	9.4	9.7	9.8	10.1	10.5	11.0	11.2	11.8	12.7	12.9
Group *B*	7.4	8.4	8.5	8.8	9.2	9.3	9.5	9.9	10.0	11.1	11.3

Jeffery was asked to test whether the two treatments resulted, on average, in a difference in growth. He chose the Wilcoxon rank sum test.

(i) Justify Jeffery's choice of test. **[1]**

(ii) Carry out the test at the 5% significance level. **[9]**

Q6, (Jun 2015, Q6)

In a two-tail Wilcoxon rank-sum test, the sample sizes are 13 and 15. The sum of the ranks for the sample of size 13 is 135. Carry out the test at the 5% level of significance. **[9]**

Q7, (Jun 2016, Q2)

Low density lipoprotein (LDL) cholesterol is known as 'bad' cholesterol.
15 randomly chosen patients, each with an LDL level of 190 mg per decilitre of blood, were given one of two treatments, chosen at random. After twelve weeks their LDL levels, in mg per decilitre, were as follows.

Treatment *A*	189	168	176	186	183	187	188	
Treatment *B*	177	179	173	180	178	170	175	174

Use a Wilcoxon rank sum test, at the 5% level of significance, to test whether the LDL levels of patients given treatment *B* are lower than the LDL levels of patients given treatment *A*. **[8]**
