Poisson Distribution (From OCR 4733)

Q1, (Jan 2006, Q1)

(i)	(a) Po(2): $1 - P(\leq 3)$ $= 0.1429$	M1 A1	**2**	Po(2) tables, "1 – " used Answer, a.r.t. 0.143
	(b) Po(2/3): $e^{-2/3}\frac{(\frac{2}{3})^2}{2!}$ $= 0.114$	M1 M1 A1	**3**	Parameter 2/3 Poisson formula correct, $r = 2$, any μ Answer, a.r.t. 0.114
(ii)	Foxes may congregate so not independent	B1 B1	**2**	Independent/not constant rate/singly used Any valid relevant application in context

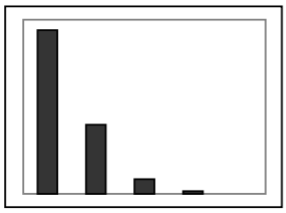
Q2, (Jan 2007, Q5i,ii)

(i)	$\lambda = 1.2$ Tables or formula used **0.6626**	B1 M1 A1	**3**	Mean 1.2 stated or implied Tables or formula [allow ± 1 term, or "1 –"] correctly used Answer in range [0.662, 0.663] [.3012, .6990, .6268 or .8795: B1M1A0]
(ii)	B(20, 0.6626√) $^{20}C_{13}\, 0.6626^{13} \times 0.3374^{7}$ **0.183**	M1 M1 A1	**3**	B(20, *p*), *p* from (i), stated or implied Correct formula for their *p* Answer, a.r.t. 0.183

Q3, (Jun 2008, Q6a)

(a)	Po(2.375) $e^{-2.375}\left(\frac{2.375^3}{3!}+\frac{2.375^4}{4!}\right)$ [= 0.2079 + 0.1233] $= \mathbf{0.3310}$	M1 M1 A1 A1	**4**	Po(19/8) stated or implied One correct Poisson formula, *not* tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]

Q4, (Jan 2009, Q3)

(i)	(a) $e^{-0.42}$ $= \mathbf{0.657}$ (b) $0.42\, e^{-0.42}$ $= \mathbf{0.276}$	M1 A1 A1	**3**	Correct formula for $R = 0$ or 1 P(0), a.r.t. 0.657 P(1), a.r.t. 0.276
(ii)	Po(2.1): $1 - P(\leq 3) = 1 - 0.8386$ $= \mathbf{0.1614}$	M1 M1 A1	**3**	Po(2.1) stated or implied Tables or formula, e.g. 0.8386 or 0.6496 or 0.9379 or complement; Answer, in range [0.161, 0.162]
(iii)		B2	**2**	At least 3 separate bars, all decreasing *Allow histogram. Allow convex* P(0) < P(1) but otherwise OK: B1 Curve: B1 *[no hint of normal allowed]*

Q5, (Jan 2010, Q9i,ii)

(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	**2**	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119
(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ [0.7720 – 0.5760] $= \mathbf{0.196}$	M1 M1 A1	**3**	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196

Q6, (Jun 2010, Q1)

(i)(a)	$1 - P(\leq 6) = 1 - 0.8675$ $= \mathbf{0.1325}$	M1 A1	**2**	1 – .9361 or 1 – .8786 or 1 – .8558: M1. .9721: M0 Or 0.132 or 0.133
(b)	Po(0.42) $e^{-0.42}\frac{0.42^2}{2!}$ $= \mathbf{0.05795}$	M1 M1 A1	**3**	Po(0.42) stated or implied Correct formula, any numerical λ Answer, art 0.058. Interpolation in tables: M1B2
(ii)	E.g. "Contagious so incidences do not occur independently", or "more cases in winter so not at constant average rate"	B2	**2**	Contextualised reason, referred to conditions: B2. No marks for mere learnt phrases or spurious reasons, e.g. not just "independently, singly and constant average rate". See notes.

Q7, (Jan 2012, Q8i-iii)

(i)		Location of bacteria must be independent – the position of one does not affect that of another	M1 A1 **[2]**	"Found independently": M1. Allow "are independent", "singly". Context needed somewhere in answer. Correct explanation, not just of "singly", e.g. not "must not group together". No extra or wrong conditions, but allow both "singly" and "independently". Right explanation, not "independent": M1A0
		Examples α Number of bacteria occurring in a particular volume is independent of the number in another interval of the same volume. Number in one volume occurs randomly. M1A0 β Bacteria are distributed independently from one another. This means that they cannot be in groups. M1A0 γ Position of each bacterium must be independent of the position of other bacteria. Not well modelled by Poisson if they tended to form groups, they must not be influenced by the surrounding bacteria or certain conditions (e,g, heat). M1A0 δ Bacteria need to be independent. The results of one cannot influence the result of another. M1A0 ε Bacteria must occur independently, so the state of one bacterium has no effect on any other bacteria. M1A0 ς Probability of bacteria must be independent, they cannot affect the probability of another bacterium occurring. M1A1 η Bacteria must occur independently, so if one occurs it can't cause more to appear. M1A1		
(ii)		$1 - P(\leq 4)$ [= 1 – 0.8912] **= 0.1088**	M1 A1 **[2]**	Allow M1 for 1 – .9580 [= 0.042] or wrong λ. 0.8912 etc: M0 0.109 or 0.1088 or better
(iii)		Po(0.925) $e^{-0.925}\frac{0.925^2}{2!}$ **= 0.169**(64)	M1 M1 A1 **[3]**	Po(0.925) stated or implied [37/40] Correct Po formula for $r = 2$, any λ, can be implied by: Answer 0.17(0) or 0.1696 or better

Q8, (Jun 2012, Q4i-iii)

(i)		Crystals must occur independently of one another	B1 [1]	Allow interpreted, or "randomly" but nothing else. Must be contextualised; no other answers included.	Ignore "singly" (meaningless in this context). But allow "probability… is independent"
(ii)		$e^{-3.2}\frac{3.2^5}{5!} = \mathbf{0.114(0)}$	M1 A1 [2]	Formula, or .0608 or .1781 or .1075 or .1203 (tables) Answer a.r.t. 0.114, implies both marks	
(iii)		Po(2.368) $1-e^{-2.368}(1+2.368+\frac{2.368^2}{2})$ **= 0.4219**	M1 M1 A1 [3]	Po(0.74 × 3.2) stated or implied 1 – correct Poisson terms, their λ, allow ± 1 term Answer, a.r.t. 0.422, implies all 3 marks	Allow for 0.75 × 3.2 etc, e.g. Po(2.4) Don't allow second M1 from λ in tables, e.g. if MR, treat as E–1. If no working: don't give M1A0

Q9, (Jun 2013, Q9)

(i)		Constant *average* rate; *or* [*] same statement *plus* "breakdowns independent" Otherwise it means that they occur at exactly regular intervals	B1 B1 **2**	State "average" or equiv, "random" or "uniform". Correct explanation	No extras apart from independence (ignore "singly") Can't get from [*]
(ii)		No because breakdowns more likely in rush hours, etc	B1 **1**	Any plausible reason for either "yes" or "no" that shows understanding of what the *statistical* concept means	Not "equally likely". *Not* reason for (in)dependence, unless [*], which needs *both* conditions if affirmed
(iii)		13 0.0739	B1 B1 **2**	0.074 or a.r.t. 0.0739. Marks independent	
(iv)		$e^{-\lambda}\frac{\lambda^2}{2!} = 0.0072$ $\lambda = \sqrt{(0.0144e^{\lambda})}$ $= 0.12e^{\lambda/2}$ $8.5 \rightarrow 8.4126$; $8.6 \rightarrow 8.8440$ Therefore solution between 8.5 and 8.6	M1* M1dep A1 A1 A1 **5**	Correct formula = their 0.0072 seen Rearrange $e^{-\lambda}$ and square root, to get $\lambda = f(\lambda)$ Correctly obtain AG, with $k = 0.5$ Two correct evaluations to 4 dp at least All completely correct and deduction stated	Allow even if left with e^{λ} or $e^{-\lambda}$ or exact equivalent 4 dp explicitly required CWO, except allow if only 3 SF

Q10, (Jun 2014, Q4)

(i)	Snakes must occur independently of one another	B1 **[1]**	Contextualised ("snakes" must be mentioned); not *just* "singly" but allow both independent and singly. Allow explanation, e.g. "Occurrence of one snake doesn't affect occurrences of others". Allow "snakes must occur randomly". Otherwise, more than one condition, "e.g. "randomly, independently, singly and at constant rate": 0.
(ii)	$1 - P(\leq 5)$ $= 1 - 0.7851 \quad = \mathbf{0.2149}$	M1 A1 **[2]**	Give M1 for 0.3712, 0.1107 or 0.2307. Answer 0.7851 is M0. Answer, a.r.t. 0.215
(iii)	Po(3.08) $e^{-3.08}\left(\frac{3.08^2}{2!}+\frac{3.08^3}{3!}\right)$ [= 0.2180 + 0.2238] $= \mathbf{0.4418}$	M1 M1 A1ft A1 **[4]**	Po(3.08) stated or implied. [Just $\lambda = 3.08$ is M0 unless Poisson later.] Correct formula for Po ($r > 0$) used at least once, can be implied Completely correct formula for their λ (not 4), can be implied Final answer, a.r.t. 0.442 No working: last 3 marks either 0 or 3, no "nearly right".

Q11, (Jun 2015, Q2)

(i)		That they don't occur regularly or to a fixed pattern, or are unpredictable	B1 **1**	Any similar or equivalent statement, but *not* independent or equivalent Both right and wrong: B0	E.g. "no pattern": expect to be right E.g. "doesn't affect": expect to be wrong
(ii)		Dead rabbits occur independently, i.e., one occurrence does not affect the probability of another *or* at constant <u>average</u> rate, i.e. mean number uniform along the whole road	B1 B1 **2**	Correct statement of principle Correct interpretation of that principle Context needed for any marks **SR:** "Constant <u>rate</u>" B0, correct reason can get B1 if "average" implied	*Not* "constant probability" One right, one wrong, e.g. independent + "$np < 5$, $nq < 5$": max 1 Only "Singly" stated, implied or used: max B1 Right condition but explanation shows it's wrong: B0B0
(iii)		Po(2.75) $e^{-2.75}\frac{2.75^3}{3!} = \mathbf{0.2215}$	M1 M1 A1 **3**	Po(1650/600) attempted Correct formula, any numerical λ Answer in range [0.221, 0.222]	Needs evidence for this <u>Must be seen</u> Formula required, so no formula ⇒ M0A0

Q12, (Jun 2016, Q4)

$\frac{\lambda^4}{4!}e^{-\lambda} = \frac{\lambda^5}{5!}e^{-\lambda}$ $\frac{\lambda^4}{4!} = \frac{\lambda^5}{5!}$	$\Rightarrow \boldsymbol{\lambda = 5}$	M1 A1 M1 A1	Poisson formula used [*not* just quoted] correctly once This equation or exact equivalent, needs $e^{-\lambda}$ seen somewhere Correct method for cancelling $e^{-\lambda}$ Solve to get $\lambda = 5$ only, www
	0.175(46)	B1 **5**	Probability, in range [0.175, 0.176], allow from $\lambda = 5$ from wrong working

Q13, (Jun 2016, Q6i-ii)

(i)	Cars pass independently of one another and at constant average rate	B1 B1	**2**	"Independently", refer to cars. Not "constant rate", "constant probability". No extra conditions. Ignore all references to "singly" (which is *wrong* in this context!)
(ii) α	$P(\leq 7) - P(\leq 3) = 0.6728 - 0.1118$ $= \mathbf{0.561}(0)$	M1 A2	**3**	0.680 or 0.681: M1A0 Allow from calculator, no working 0.4491 or 0.5679: M1A1 Allow from calculator, no working
or β	$P(4) + P(5) + P(6) + P(7)$ $= 0.1118 + 0.1454 + 0.1575 + 0.1462$ $= \mathbf{0.561}(0)$	M1 A1 A1	**3**	Correct formula for ≥ 3 probabilities from Po(6.5) added, can be implied 3, 4 or 5 correct terms (e.g. P(3) = 0.06880), can be algebraic or implied Answer, a.r.t. 0.561