Hypothesis Tests Requiring Unbiased Estimators and Central Limit Theorem (From OCR 4733)

Q1, (Jan 2006, Q6)

H_0: $\mu = 32$; H_1: $\mu > 32$, where μ is population mean waist measurement $\overline{W} = 32.3$ $s^2 = 52214.50/50 - \overline{W}^2$ [= 1] $\hat{\sigma}^2 = 50/49 \times s^2$ [= 50/49 or 1.0204]	B1 B1 B1 M1 M1	One hypothesis correctly stated, *not* x or $\bar{x}$ or $\overline{w}$ Both completely correct, μ used Sample mean 32.3 seen Correct formula for s^2 used Multiply by 50/49 or √
α: $z = (32.3 - 32)\times\sqrt{49}$ $= 2.1$ Compare 2.1 with 3.09 or 0.0179 with 0.001	M1 A1 B1	Correct formula for z, can use s, aef, need $\mu = 32$ $z = 2.1$ or $1 - \Phi(z) = 0.0179$, *not* –2.1 Explicitly compare their 2.1 with 3.09(0) or their 0.0179 with 0.001
β: CV $= 32 + 3.09 \div\sqrt{49}$ $= 32.44$ Compare CV with 32.3	M1 B1 A1√	$32 + z \times \sigma/\sqrt{n}$ [allow ±, s, any z] $z = 3.09$ and (later) compare $\bar{x}$ CV in range [32.4, 32.5], √ on k
Do not reject H_0 Insufficient evidence that waists are actually larger	M1√ A1√ **10**	Correct conclusion, can be implied, needs essentially correct method including $\sqrt{n}$, any reasonable σ, but not from $\mu = 32.3$ Interpreted in context

Q2, (Jun 2008, Q3)

α	$H_0: \mu = 28$ $H_1: \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38] $z = \dfrac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare –1.645, or 0.0547 with 0.05	B2 M1 M1 A1 B1	Both hypotheses correctly stated; one error, allow wrong or no letter, but not x or t or $\bar{x}$, B1 Multiply 37.05 or √37.05 by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$ Standardise with $\sqrt{n}$, allow √ errors, cc, + Correct z, a.r.t –1.60, or $p \in [0.0547, 0.0548]$ Explicit comparison of z with –1.645 or p with 0.05
β	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare 26.44 with 26.40	M1 B1 A1√	Allow "±", √ errors, cc, ignore other tail $z = 1.645$ in CV expression, and compare 26.44 CV, √ on their z, rounding to 3 SF correct
	Do not reject H_0 [can be implied] Insufficient evidence that time taken has changed.	M1 A1√ **8**	Needs $\sqrt{n}$, correct method & comparison, *not* $\mu = 26.44$ Conclusion interpreted in context, √ on z,

Q3, (Jan 2009, Q7)

(i)	$\hat{\mu} = \bar{t} = 13.7$ $\dfrac{12657.28}{64} - 13.7^2$ [=10.08]; $\times \frac{64}{63}$ **= 10.24** $H_0: \mu = 13.1$, $H_1: \mu > 13.1$ $\dfrac{13.7 - 13.1}{\sqrt{10.24/64}} = 1.5$ or $p = 0.0668$ 1.5 < 1.645 or 0.0668 > 0.05 Do not reject H_0. Insufficient evidence that time taken on average is greater than 13.1 min	B1 M1 M1 A1 B2 M1 A1 B1 M1 A1 **11**	13.7 stated Correct formula for biased estimate $\times \frac{64}{63}$ used, or equivalent, can come in later Variance or SD 10.24 or 10.2 Both correct. [SR: One error, B1, but x or t or $\bar{x}$ or $\bar{t}$, 0] Standardise, or find CV, with √64 or 64 z = a.r.t. 1.50, or p = 0.0668, or CV 13.758 [√ on z] Compare z & 1.645, or p & 0.05 (must be correct tail), or z = 1.645 & 13 with CV Correct comparison & conclusion, needs 64, *not* $\mu = 13.7$ Contextualised, some acknowledgement of uncertainty [13.1 – 13.7: (6), M1 A0 B1 M0]
(ii)	Yes, not told that dist is normal	B1 **1**	Equivalent statement, *not* "n is large", don't need "yes"

Q4, (Jan 2011, Q4)

(i)	*Either* $z = \frac{213.4 - 230}{45/\sqrt{50}}$ $= -2.608$ $-2.608 < -2.576$ *or* $0.0047 < 0.005$	M1 A1 B1	Standardise z with √50, ignore sign or √ or squaring errors z-value, a.r.t. –2.61, or p in range [0.0044, 0.005) Correctly compare (–)2.576, signs consistent, *or p* explicitly with 0.005
Or	CV is $230 - 2.576 \times \frac{45}{\sqrt{50}} = 213.6$ $213.4 < 213.6$	M1 B1 A1	$230 - z\sigma/\sqrt{50}$, allow √ or squaring errors, allow ± but not just +; $z = 2.576$ Explicitly compare 213.4 with 213.6
	Reject H_0. Significant evidence that population mean is not 230	M1 A1 FT **5**	"Reject", FT, needs correct method and form of comparison; interpreted, acknowledge uncertainty
(ii)	Yes, population distribution is not known to be normal	B2 **2**	*Not*, "yes, sample size is large" but ignore "*can* use it as …" SR: Both right and wrong answers: B1 α "Yes as it must be assumed normal": B1

Q5, (Jun 2011, Q6)

(i)	H_0: $\mu = 24.3$; H_1: $\mu \neq 24.3$	B1B1	Both: B2. 1 error, B1, but *t*, *x* etc: B0 ***SEE NOTES AT START AND END***
	$\bar{t} = 26.28$	B1	26.28 seen or implied
	$\hat{\sigma}^2 = \frac{50}{49}\left[\frac{36602.17}{50} - 26.28^2\right]$	M1 M1	Correct formula for biased estimate [= 41.405] Multiply by 50/49 [Single formula: M2, or give M1 if wrong but 49 divisor seen]
	$= 42.25$	A1	42.25 or 6.5 seen or implied
α	$z = \frac{26.28 - 24.3}{\sqrt{42.25/50}} = 2.154$	M1 A1	Standardise their $\bar{t}$ with 24.3, √50, allow sign/√/cc errors, their variance 2.15(4) *or p* in range [0.0153, 0.0158], *not* –2.154 unless 0.015(6) subsequently used, *not* 1-tail
	< 2.576	A1	Compare *z* with ±2.576, or $p > 0.005$, or 2*p* with 0.01, *not* from $\mu = 26.28$
β	CV $24.3 + 2.576 \times \sqrt{\frac{42.25}{50}}$	M1 A1	$24.3 + zs/\sqrt{50}$, allow cc, √ errors, allow ± but not – only. Not $26.28 - zs/\sqrt{50}$ $z = 2.576$, *not* from $\mu = 26.28$ or 50 omitted, *not* from 1-tail
	$= 26.67$ and $26.28 < 26.67$	A1	Correct CV, ✓ on *z*, and compare sample mean
	Do not reject H_0. Insufficient evidence of a change in maximum daily temperature.	M1 A1✓ **11**	Conclusion, ✓, needs method, like-with-like, 50, *not* from $\mu = 26.28$, *doesn't* need correct *z* Contextualised, recognise uncertainty, ✓ on numbers NB: Clear evidence of $\mu = 26.28$: can't get last 4 marks. *See exemplars γ and δ*
(ii)	*n* is large	B1 **1**	This answer ***only*** or "*n* > number" where number ≥ 29, *not* both this and "distribution unknown". But "*n* is large so we can approximate even though we don't know the distribution" is B1 "Possible as *n* = 50" B0.

Q6, (Jun 2012, Q5)

α:	$H_0: \mu = 6.1$ $H_1: \mu \neq 6.1$ $\hat{\mu} = \bar{x} = 6.2$ $\hat{\sigma}^2 = \frac{80}{79}\left(\frac{3126}{80} - 6.2^2\right) = 0.643$ $z = \frac{6.2 - 6.1}{\sqrt{0.643/80}} = 1.115$ [1 – Φ(1.115) = 0.1325>0.05] $1.115 < 1.645$	B2 B1 M1 M1 A1 M1 A1 A1	Both: B2. One error, B1, but $\bar{x}$, x, r etc: 0. 6.2: B0 6.2 [31/5] seen somewhere (other than hypotheses) Correct formula for biased estimate [0.635 or 127/200] Divide by 79 somewhere Variance estimate, a.r.t. 0.643, can be implied Standardise their 6.2 with reasonable variance attempt, needs 80, allow cc $z \in [1.11, 1.12]$ (not –) or $p \in [0.1323, 0.1333]$ Compare z with 1.645 (allow –1.645 if $z < 0$) or p *(< 0.5)* with 0.05	If single formula used, M2 or, if wrong, allow M1 for divisor 79 anywhere [254/395 leading to 127/15800] *80 needed, otherwise M0 and no more marks* *If clearly $\mu = 6.2$ used, no more marks* A1 uses number used for comparison Withhold if inequality incorrect or if 1-tailed Must be consistent signs/tails and like-with-like
β:	CV $6.1 + 1.645 \times \sqrt{\frac{0.643}{80}}$ $= 6.247$ and $6.2 < 6.247$	M1 A1 A1√	$6.1 + z\sqrt{(\sigma^2/80)}$, allow ±, √ errors CV, a.r.t. 6.25, needs $z = 1.645$, allow biased $\hat{\sigma}^2$ Compare 6.2 with CV from + sign, √ on z (but not σ)	Allow 6.2 – (or ±) but no more marks afterwards If no 79 earlier but used here, recovers M1A1 E.g. 1.96 → 6.276 or 1.282 → 6.215 [gets M1A0A1
	Do not reject H_0. Insufficient evidence that pH value is not 6.1	M1 A1√ [11]	Needs essentially correct method and comparison, needs 80 but no need for correct variance Needs context and "evidence" or equivalent, ft on their z/p/CV	First conclusion wrong: M0A0 even if second correct. "1.115 > 1.645 so do not reject H_0" etc: (A0)M1A1
	Biased estimate used : typically gets B2B1 M1M0A0 M1A0A1 M1A1 [total 8]		$\bar{x}$ and μ interchanged: allow final M1A1 if *anywhere* right, but if always wrong (in hypotheses and z) M0A0. This would typically get B0B0B1 M1M1A1 M1A0A0 M0A0 [total 5]	

Q7, (Jun 2014, Q7)

(i)	$\hat{\mu} = \bar{x} = 81$	B1	81 only, can be implied
	$\frac{329800}{50} - 81^2$ [= 35]	M1	Correct formula for biased estimate, their "81", can be implied
	$\times \frac{50}{49}$; $= 35.71$	M1	Multiply by 50/49. SC: single formula: M2, or M1 if wrong but divisor 49 anywhere [can be recovered if correctly done in part (ii)]
		A1	A.r.t. 35.7 – can't be recovered from part (ii). Can be implied
	$1 - \Phi\left(\frac{90-81}{\sqrt{35.71}}\right) = 1 - \Phi(1.506) = 1 - 0.9339$	M1	Standardise with their μ and σ, allow σ^2, cc but not $\sqrt{50}$
	= 6.61% or **0.0661**	A1 **[6]**	Answer, a.r.t. 6.6% or 0.066

(ii)	H_0: $\mu = 80$ H_1: $\mu \neq 80$	B2	Correct, B2. One error, e.g. wrong or no symbol, >, B1, but x or $\bar{x}$ or t etc, or 81, B0. NB: If both hypotheses involve 81, *can't* get final M1
α:	$z = \frac{81-80}{\sqrt{35.71/50}} = 1.183$ [*or* $p = 0.1183$]	M1 A1	Standardise, with √50, allow √, sign or cc errors, allow from biased variance z, a.r.t. 1.18, or p, a.r.t. 0.118. Allow –1.18.
	< 1.645	B1	Their $z < 1.645$ or $p > 0.05$, *not* if one-tail. Allow $-1.18 > -1.645$. *Not* just 1.645 seen.
β:	CV $80 + 1.645\sqrt{\frac{35.71}{50}} = 81.39$ $81 < 81.39$	M1 B1 A1	80 + zs/√50, allow √ or cc errors, ignore – (no marks for – alone); $z = 1.645$ used in this expression (not just seen), *not* from one-tail Compare CV with 81, allow 81.08 from one-tailed ($z = 1.282$) (but not on their σ) SC: $81 - 1.645\sqrt{\frac{35.71}{50}}$: If H_0: $\mu = 80$: (B2) M1B1A0M0A0. If H_0: $\mu = 81$: (B0) M1B1A1 (79.61) M0A0
	Do not reject H_0. Insufficient evidence that the mean time is not 80 minutes.	M1 A1ft **[7]**	Correct first conclusion, needs √50, correct comparison type, μ and $\bar{x}$ not consistently wrong way round (thus H_0: $\mu = 81$ can get B0 M1A1A1 M0A0, max 3/7) In method β, it needs to be clear that comparison involves $\bar{x}$. Contextualised (mention "time"), acknowledge uncertainty ("evidence that…") *Not* "significant evidence that mean time is 80" FT on wrong z-value or wrong critical value if previous mark gained SC: One-tailed: can get B1B0 M1A1B0 M1A1, max 5/7 No √50: can get B2 M0 B1 M0, max 3/7
(iii)	(a) Yes (single observation only) (b) No, CLT applies to large sample	B1 B1 **[2]**	No reason needed, but withhold if wrong reason seen. Allow "yes, no dist^n given" "No" *and* refer to central limit theorem or "large sample" {note for scoris zoning – (a) and (b) to be in single zone}

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Q8, (Jun 2015, Q6)

(i)		$\bar{t} = 11.76$ $\hat{\sigma}^2 = \frac{120}{119}\left(\frac{18737.712}{120} - 11.76^2\right) = 18$ $H_0: \mu = 11.0$, $H_1: \mu \neq 11.0$ α: $z = \frac{11.76 - 11.0}{\sqrt{18/120}} = \mathbf{1.9623}$ > 1.645	B1 M1 M1 A1 B2 M1 A1 A1	11.76 seen or implied Biased estimate (= 17.85) ×120/119, *or* single formula with 119 divisor Answer 18 ± 0.05 One error, B1, but $\bar{t}$, t, x etc: B0 (u: B1) Standardise with 120, ignore cc or √ errors A.r.t. (±)1.96 *or* $p \in [0.0245, 0.025]$ www Compare explicitly with (±)1.645 or 0.05, consistent with their z or p. *[Needs to be "next to" TS]*	i.e. correct single formula gets M2 If both hypotheses involve 11.76, only further mark possible is next M1 [max 5/11] 120 omitted gets no further marks [max 6/11] Ignore "N(11.76, …)" *unless* hypotheses omitted altogether, in which case treat as hypotheses in terms of 11.76
		β: CV $11.0 \pm 1.645\times\sqrt{(18/120)}$ = 11.637 (or 10.363) 11.76 > 11.64	M1 A1 A1	$11.0 + z\sigma/\sqrt{120}$, needs 120 and + or ± Ignore 10.363 Explicit comparison, consistent tail	If $11.76 - z\sigma/\sqrt{120}$, give M1A0A0 M0A0 (even if correct hypotheses)
		Reject H_0. Significant evidence that the average time has changed.	M1 A1ft **11**	Correct first conclusion, allow "Accept H_1" Contextualised, acknowledge uncertainly, FT on wrong CR/z/p	Needs correct method (including 120) and comparison type, 11.0 in at least one hypothesis Allow "increase" instead of "change"
(ii)		No, the Central Limit Theorem applies	B1 **1**	or "No, large sample". Withhold if extra wrong or irrelevant reason(s) given	Needs both "no" and reason

Q9, (Jun 2016, Q8)

(i) α:	H_0: $\mu = 13.3$, H_1: $\mu < 13.3$ $z = \dfrac{12.48 - 13.3}{\sqrt{12.25/50}} = -1.6566$ [$p = 0.0488$] [12.25/50 = 0.245] < -1.645 [$p < 0.05$]	B2 M1 A1 B1	Both correct: B2. One error [e.g. p, ≠, no symbol] B1, but x, $\bar{x}$ etc B0 Standardise with √50, allow √ errors, allow cc, allow 13.3 – 12.48 z in range [–1.66, –1.65], or p in range [0.04875, 0.0489], allow 0.9512 only if consistent Compare with –1.645, allow +1.6566 with +1.645, or p with 0.05/0.95 as consistent
β:	CV $13.3 - 1.645\sqrt{\dfrac{12.25}{50}} = 12.4857\ldots$ 12.48 < CV	M1 B1 A1	$13.3 - z\sigma/\sqrt{50}$, any recognisable z, allow √ errors etc, ignore 13.3 + … $z = 1.645$ Compare 12.49 (or better) with 12.48, ignore 13.3 + … SC: 2-tailed, 12.33 gets B1B0 M1B0A1ft M1A1
	Reject H_0. Significant evidence that animals in zoos have shorter expected lifetime	M1 A1ft **7**	Consistent, needs √50, like-with-like comparison, hypotheses *not* 12.48 Contextualised, acknowledge uncertainty, their z SC1: 2-tailed: can get B1 M1A1B0 M1A1 max 5/7 SC2: No √50: can get B2 M0A0 B1 M0 max 3/7 SC3: $\bar{x}$ and μ confused consistently: can get B0 M1A1 B1 M0 SC4: 50/49 used in (i): can get B2 M1A0B1 M1A1 (6) in (i), M1 in (ii)
(ii)	$\hat{\sigma}^2 = \dfrac{50}{49} \times 12.25$ [= 12.5] $z = \dfrac{12.48 - 13.3}{\sqrt{12.5/50}} = -1.64$ [$p = 0.0505$] > -1.645 [$p > 0.05$] Opposite conclusion	M1 M1 A1 B1 A1ft **5**	Multiply 12.25 by 50/49, allow √ etc, allow if done in part (i) but then 0 Standardise with √50 Obtain a.r.t. –1.64, allow +1.64 if consistent with (i). Compare with same CV as in (i) State opposite conclusion (ft), any form, allow $\bar{x}/\mu$ here , needs M1M1 *Identical mark scheme for method β, CV 12.4775* SC1: 50 omitted consistently in both: M1M0A0B1A1 max 3/5 SC2: no √50 in (i), √50 but not 50/49 in (ii): M0M1A0B1A1 max 3/5
(iii)	Yes as population not known to be normal	B1 **1**	Not "n large" unless "Yes, not known normal, but n large so can use" No wrong extras, e.g. "depends on whether it's sample or population"