Hypothesis Tests for Correlation (From OCR 4767)

Q1, (Jan 2006, Q3)

<table>
<tr><td>(i)</td><td>
<table>
<tr><td>Rank x</td><td>1</td><td>5</td><td>4</td><td>7</td><td>6</td><td>8</td><td>10</td><td>3</td><td>9</td><td>2</td></tr>
<tr><td>Rank y</td><td>2</td><td>4</td><td>5</td><td>8</td><td>9</td><td>7</td><td>10</td><td>6</td><td>3</td><td>1</td></tr>
<tr><td>d</td><td>-1</td><td>1</td><td>-1</td><td>-1</td><td>-3</td><td>1</td><td>0</td><td>-3</td><td>6</td><td>1</td></tr>
<tr><td>d^2</td><td>1</td><td>1</td><td>1</td><td>1</td><td>9</td><td>1</td><td>0</td><td>9</td><td>36</td><td>1</td></tr>
</table>

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 60}{10 \times 99}$$

= 0.636 (to 3 s.f.) [*allow* 0.64 to 2 s.f.]</td><td>M1 for ranking (allow all ranks reversed)
M1 for d^2
A1 CAO for Σd^2
M1 for structure of r_s using their Σd^2
A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td><td>5</td></tr>
<tr><td>(ii)</td><td>H$_0$: no association between x and y
H$_1$: positive association between x and y
Looking for positive association (one-tail test):
Critical value at 5% level is 0.5636
Since 0.636 > 0.5636, there is sufficient evidence to reject H$_0$,
i.e. conclude that there appears to be positive association between temperature and nitrous oxide level.</td><td>B1 for H$_0$
B1 for H$_1$
NB H$_0$ H$_1$ <u>not</u> ito rho
B1 for ± 0.5636
(FT their H$_1$)
M1 for comparison with c.v., provided $|r_s| < 1$
A1 for conclusion in words f.t. their r_s and sensible cv</td><td>5</td></tr>
<tr><td>(iii)</td><td>Underlying distribution must be bivariate normal.
If the distribution is bivariate normal then the scatter diagram will have an elliptical shape.
This scatter diagram is not elliptical and so a PMCC test would not be valid.
(Allow comment indicating that the sample is too small to draw a firm conclusion on ellipticity and so on validity)</td><td>B1 CAO for bivariate normal
B1 indep for elliptical shape
E1 dep for conclusion</td><td>3</td></tr>
<tr><td>(iv)</td><td>n=60, PMCC critical value is $r = 0.2997$
So the critical region is $r \geq 0.2997$</td><td>B1
B1 FT their sensible c.v.</td><td>2</td></tr>
<tr><td>(v)</td><td>Any three of the following:
Correlation does not imply causation;
There could be a third factor (causing the correlation between temperature and ozone level);
the claim could be true;
increased ozone could cause higher temperatures.</td><td>E1
E1
E1</td><td>3</td></tr>
<tr><td></td><td></td><td></td><td>18</td></tr>
</table>

Q2, (Jun 2006, Q3)

(i)	**EITHER:** $S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x\Sigma y = 6235575 - \frac{1}{10} \times 4715 \times 13175$ $= 23562.5$ $S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 2237725 - \frac{1}{10} \times 4715^2 =$ 14602.5 $S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 17455825 - \frac{1}{10} \times 13175^2 =$ 97762.5 $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{23562.5}{\sqrt{14602.5 \times 97762.5}} = 0.624$ **OR:** $\text{cov}(x,y) = \frac{\sum xy}{n} - \overline{xy} = 6235575/10 - 471.5 \times 1317.5$ $= 2356.25$ $\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(14602.5/10)} = \sqrt{1460.25} = 38.21$ $\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(97762.5/10)} = \sqrt{9776.25} = 98.87$ $r = \frac{\text{cov(x,y)}}{rmsd(x)\,rmsd(y)} = \frac{2356.25}{38.21 \times 98.87} = 0.624$	M1 for method for S_{xy} M1 for method for at least one of S_{xx} or S_{yy} A1 for at least one of S_{xy}, S_{xx} or S_{yy} correct M1 for structure of *r* A1 (0.62 to 0.63) M1 for method for cov (*x*,*y*) M1 for method for at least one msd A1 for at least one of S_{xy}, S_{xx} or S_{yy} correct M1 for structure of *r* A1 (0.62 to 0.63)	**5**
(ii)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ (two-tailed test) where ρ is the population correlation coefficient For $n = 10$, 5% critical value = 0.6319 Since 0.624 < 0.6319 we cannot reject H_0: There is not sufficient evidence at the 5% level to suggest that there is any correlation between length and circumference.	B1 for H_0, H_1 in symbols B1 for defining ρ B1FT for critical value M1 for sensible comparison leading to a conclusion A1 FT for result B1 FT for conclusion in context	**6**
(iii)	*(A)* This is the probability of rejecting H_0 when it is in fact true. *(B)* Advantage of 1% level – less likely to reject H_0 when it is true. Disadvantage of 1% level – less likely to accept H_1 when H_0 is false.	B1 for 'P(reject H_0)' B1 for 'when true' B1, B1 Accept answers in context	**2** **2**

(iv)	The student's approach is not valid. If a statistical procedure is repeated with a new sample, we should not simply ignore one of the two outcomes. The student could combine the two sets of data into a single set of twenty measurements.	E1 E1 – allow suitable alternatives. E1 for combining samples.	**3**
			18

Q3, (Jun 2007, Q2)

<table>
<tr>
<td>(i)</td>
<td>
<table>
<tr><td>x</td><td>2.61</td><td>2.73</td><td>2.87</td><td>2.96</td><td>3.05</td><td>3.14</td><td>3.17</td><td>3.24</td><td>3.76</td><td>4.1</td></tr>
<tr><td>y</td><td>3.2</td><td>2.6</td><td>3.5</td><td>3.1</td><td>2.8</td><td>2.7</td><td>3.4</td><td>3.3</td><td>4.4</td><td>4.1</td></tr>
<tr><td>Rank x</td><td>10</td><td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td></tr>
<tr><td>Rank y</td><td>6</td><td>10</td><td>3</td><td>7</td><td>8</td><td>9</td><td>4</td><td>5</td><td>1</td><td>2</td></tr>
<tr><td>d</td><td>4</td><td>-1</td><td>5</td><td>0</td><td>-2</td><td>-4</td><td>0</td><td>-2</td><td>1</td><td>-1</td></tr>
<tr><td>d^2</td><td>16</td><td>1</td><td>25</td><td>0</td><td>4</td><td>16</td><td>0</td><td>4</td><td>1</td><td>1</td></tr>
</table>
$r_s = 1 - \dfrac{6\Sigma d^2}{n(n^2-1)} = 1 - \dfrac{6 \times 68}{10 \times 99}$
= 0.588 (to 3 s.f.) [*allow* 0.59 to 2 s.f.]
</td>
<td>M1 for ranking (allow all ranks reversed)

M1 for d^2

A1 for $\Sigma d^2 = 68$

M1 for method for r_s
A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td>
<td>5</td>
</tr>
<tr>
<td>(ii)</td>
<td>H$_0$: no association between x and y
H$_1$: positive association between x and y
Looking for positive association (one–tail test): critical value at 5% level is 0.5636
Since 0.588> 0.5636, there is sufficient evidence to reject H$_0$,
i.e. conclude that there is positive association between true weight x and estimated weight y.</td>
<td>B1 for H$_0$ in context.
B1 for H$_1$ in context.
NB H$_0$ H$_1$ <u>not</u> ito ρ
B1 for ± 0.5636
M1 for sensible comparison with c.v., provided $|r_s| < 1$
A1 for conclusion in words & in context, f.t. their r_s and sensible cv</td>
<td>5</td>
</tr>
<tr>
<td>(iii)</td>
<td>$\Sigma x = 31.63,\ \Sigma y = 33.1,\ \Sigma x^2 = 101.92,\ \Sigma y^2 = 112.61,$
$\Sigma xy = 106.51.$
$S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x \Sigma y = 106.51 - \frac{1}{10} \times 31.63 \times 33.1$
$= 1.8147$
$S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 101.92 - \frac{1}{10} \times 31.63^2 = 1.8743$
$S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 112.61 - \frac{1}{10} \times 33.1^2 = 3.049$
$r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \dfrac{1.8147}{\sqrt{1.8743 \times 3.049}} = 0.759$</td>
<td>M1 for method for S_{xy}

M1 for method for at least one of S_{xx} or S_{yy}

A1 for at least one of S_{xy}, S_{xx}, S_{yy} correct.

M1 for structure of r

A1 (awrt 0.76)</td>
<td>5</td>
</tr>
<tr>
<td>(iv)</td>
<td><i>Use of the PMCC is better since it takes into account not just the ranking but the actual value of the weights.</i> Thus it has more information than Spearman's and will therefore provide a more discriminatory test.

Critical value for rho = 0.5494
PMCC is very highly significant whereas Spearman's is only just significant.</td>
<td>E1 for has values, not just ranks
E1 for contains more information
Allow alternatives.
B1 for a cv
E1 dep</td>
<td>4</td>
</tr>
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<td>19</td>
</tr>
</table>

Q4, (Jan 2009, Q1i,ii)

(i)

x	18	43	52	94	98	206	784	1530
y	1.15	0.97	1.26	1.35	1.28	1.42	1.32	1.64
Rank x	1	2	3	4	5	6	7	8
Rank y	2	1	3	6	4	7	5	8
d	-1	1	0	-2	1	-1	2	0
d^2	1	1	0	4	1	1	4	0

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6\times 12}{8\times 63}$$

= 0.857 (to 3 s.f.) [*allow* 0.86 to 2 s.f.]

M1 for attempt at ranking (allow all ranks reversed)

M1 for d^2

A1 for $\Sigma d^2 = 12$

M1 for method for r_s

A1 f.t. for $|r_s| < 1$

NB No ranking scores zero

5

(ii)

H_0: no association between X and Y in the population

H_1: some association between X and Y in the population

Two tail test critical value at 5% level is 0.7381

Since 0.857> 0. 7381, there is sufficient evidence to reject H_0,
i.e. conclude that the evidence suggests that there is association between population size X and average walking speed Y.

B1 for H_0

B1 for H_1

B1 for population SOI

NB H_0 H_1 <u>not</u> ito ρ

B1 for ± 0. 7381

M1 for sensible comparison with c.v., provided $|r_s| < 1$

A1 for conclusion in words f.t. their r_s and sensible cv

6

Q5, (Jan 2011, Q1)

(i)	**EITHER:**			
	$S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x\Sigma y = 1398.56 - \frac{1}{14} \times 139.8 \times 140.4$ $= -3.434$	M1 for method for S_{xy}		
	$S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 1411.66 - \frac{1}{14} \times 139.8^2 = 15.657$	M1 for method for at least one of S_{xx} or S_{yy}		
	$S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 1417.88 - \frac{1}{14} \times 140.4^2 = 9.869$	A1 for at least one of S_{xy}, S_{xx}, S_{yy} correct		If $\bar{x}$ and $\bar{y}$ used in rounded form, be generous with first A1
	$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{-3.434}{\sqrt{15.657 \times 9.869}}$	M1 for structure of r		Structure of r needs to be fully correct in all parts – the first two M1 marks must have been earned and $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ applied.
	$= -0.276$	A1 (-0.27 to -0.28 to 2dp)		
	OR:			
	$\text{cov}(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = 1398.56/14 - 9.9857 \times 10.0286$ $= -0.2454$	M1 for method for cov (x,y)		
	$\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(15.657/14)} = \sqrt{1.1184} = 1.0575$	M1 for method for at least one msd		
	$\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(9.869/14)} = \sqrt{0.7049} = 0.8396$	A1 for at least one of cov (x,y), msd(x), msd(y) correct		If $\bar{x}$ and $\bar{y}$ used in rounded form, be generous with first A1
	$r = \frac{\text{cov(x,y)}}{rmsd(x)\,rmsd(y)} = \frac{-0.2454}{1.0575 \times 0.8396}$	M1 for structure of r		
	$= -0.276$ NB: using only 3dp in calculating $\bar{x}$ and $\bar{y}$ leads to answer of 0.284 which is still in the acceptable range	A1 (-0.27 to -0.28 to 2dp)	**5**	Structure of r needs to be fully correct in all parts – the first two M1 marks must have been earned and $r = \frac{\text{cov(x,y)}}{rmsd(x)\,rmsd(y)}$ applied.

(ii)	H_0: $\rho = 0$ H_1: $\rho \neq 0$ (two-tailed test) where ρ is the population correlation coefficient For $n = 14$, 5% critical value = -0.5324 Since $-0.276 > -0.5324$ the result is not significant. Thus we do not have sufficient evidence to reject H_0 There is not sufficient evidence at the 5% level to suggest that there is correlation between birth rate and death rate	B1 for H_0, H_1 in symbols B1 for defining ρ B1 for critical value (+ or -) M1 for a sensible comparison leading to a conclusion (provided that $-1 < r < 1$) A1 for correct result ft their r B1 ft for conclusion in context	**6**	Condone hypotheses written in words and context. e.g. allow H_0: There is no correlation between x & y, H_1: There is correlation between x & y. (i.e. allow x & y as 'context' since these are defined in the question) NB If hypotheses given only in words and 'association' mentioned then do not award first B1 and last B1 For hypotheses written in words, candidates must make it clear that they are testing for evidence of correlation in the population. One-tailed test cv = (-) 0.4575 Comparison should be between the candidate's value of r from part (i) and an appropriate cv (i.e. the sign of the cv and the sign of r should be the same). NOTE If result not stated but final conclusion is correct award SC1 to replace the final A1 B1
(iii)	The underlying population must have a bivariate Normal distribution. Since the scatter diagram has a roughly elliptical shape.	B1 E1 for elliptical shape	**2**	Not bivariate and Normal
(iv)	Because this data point is a long way from the other data and it is below and to the right of the other data. It does bring the validity of the test into question since this extra data point is so far from the other points and so there is less evidence of ellipticity.	E1 for a long way E1 for below and to the right of. E1 for does cast doubt on validity E1 for less elliptical	**4**	Indication that the point is (possibly) an outlier For identifying the position of this point (allow in terms of x and y) Allow 'no' but only with with suitable explanation e.g. the sample is still too small to provide evidence either for or against the presence of ellipticity.
		TOTAL	**17**	

Q6, (Jun 2012, Q1)

(i)		**EITHER**			
		$S_{xy} = \Sigma xy - \frac{1}{n}\Sigma x\Sigma y = 600.41 - \frac{1}{10}\times 113.69 \times 52.81 = 0.01311$	M1	For method for S_{xy}	
		$S_{xx} = \Sigma x^2 - \frac{1}{n}(\Sigma x)^2 = 1292.56 - \frac{1}{10}\times 113.69^2 = 0.01839$	M1	For method for at least one of S_{xx} or S_{yy}	
		$S_{yy} = \Sigma y^2 - \frac{1}{n}(\Sigma y)^2 = 278.91 - \frac{1}{10}\times 52.81^2 = 0.02039$	A1	For at least one of S_{xy}, S_{xx} or S_{yy} correct	
		$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{0.01311}{\sqrt{0.01839\times 0.02039}} = 0.677$	M1 A1	For fully correct structure of r For answer rounding to 0.68	
		OR			
		$\text{cov}(x,y) = \frac{\sum xy}{n} - \overline{xy} = 600.41/10 - 11.369\times 5.281 = 0.001311$	M1	For method for cov (x,y)	
		$\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(0.01839/10)} = \sqrt{0.001839} = 0.04288$	M1	For method for at least one msd or rmsd	
		$\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(0.02039/10)} = \sqrt{0.002039} = 0.04516$	A1	For at least one of cov (x,y), msd or rmsd correct	
		$r = \frac{\text{cov(x,y)}}{rmsd(x)\,rmsd(y)} = \frac{0.001311}{0.04288\times 0.04516} = 0.677$	M1 A1	For fully correct structure of r For answer rounding to 0.68	
				Methods mixed – max M0M1A1M0A0	
			[5]		
(ii)		H_0: $\rho = 0$ H_1: $\rho \neq 0$ (two-tailed test)	B1	For H_0, H_1 in symbols. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population correlation coefficient.	
		where ρ is the population correlation coefficient	B1	For defining ρ. Condone omission of "population" if correct notation ρ is used, but if ρ is defined as the **sample** correlation coefficient then award **B0**.	
		For $n = 10$, 10% critical value = 0.5494	B1	CAO Note that critical values for a one-tailed test at the 10% level are not available in tables.	

		Since 0.677 > 0.5494 the result is significant.	M1	For sensible comparison leading to a conclusion provided that $\|r\| < 1$. The comparison can be in the form of a diagram as long as it is clear and unambiguous. Sensible comparison: e.g. 0.677 > 0.5494 is 'sensible' whereas 0.677 > - 0.5494 is 'not sensible'. Reversed inequality sign e.g. 0.677 < 0.5494 etc. gets max M1 A0.	
		(Thus we have sufficient evidence to) reject H_0	A1*	For reject H_0 o.e. FT their *r* and critical value from 10% 2-tail column.	
		There is sufficient evidence at the 10% level to suggest that there is correlation between times for the first and last sections.	E1dep* [6]	For correct, non-assertive conclusion in context. Allow '*x* and *y*' for context. E0 if H_0 and H_1 not stated, reversed or mention a value other than zero for ρ in H_0. Do not allow 'positive correlation' or 'association'	
(iii)		The underlying population must have a bivariate Normal distribution.	B1	Condone "bivariate Normal distribution", "underlying bivariate Normal distribution", but **do not allow** "the **data** have a bivariate Normal distribution"	
		The points in the scatter diagram should have a roughly elliptical shape.	E1 [2]	Condone 'oval' or suitable diagram	
(iv)		The hypothesis test has shown that there appears to be correlation.	E1	For relevant comment relating to the test result or positive value of *r* in supporting (unless FT leads to not supporting) the commentator's suggestion. Or correlation does not imply causation. There may be a third factor. For questioning the use of the word 'must'	
		However it could be that there is a third causal factor	E1 [2]	Allow any two suitable, statistically based comments.	
(v)	(*A*)	Yes because the critical value at the 1% level is 0.7646 which is larger than the test statistic	B1* E1dep* [2]	B1 for 0.7646 seen E1 for comment consistent with their (ii) provided $r\| < 1$	
(v)	(*B*)	One advantage of a 1% level is that one is less likely to reject the null hypothesis when it is true.	E1	o.e. Wording must be clear.	
		One disadvantage of a 1% level is that one is more likely to accept the null hypothesis when it is false.	E1 [2]	o.e.	

Q7, (Jun 2013, Q1)

(i)		**EITHER:**		
		$S_{xy} = \sum xy - \frac{1}{n}\sum x\sum y = 40.66 - \frac{1}{60} \times 43.62 \times 55.15$ $= 0.56595$	M1*	For method for S_{xy}
		$S_{xx} = \sum x^2 - \frac{1}{n}\left(\sum x\right)^2 = 32.68 - \frac{1}{60} \times 43.62^2$ $= 0.96826$	M1*	For method for at least one of S_{xx} or S_{yy}
		$S_{yy} = \sum y^2 - \frac{1}{n}\left(\sum y\right)^2 = 51.44 - \frac{1}{60} \times 55.15^2$ $= 0.74796$	A1	For at least one of S_{xy}, S_{xx} or S_{yy} (to 2 sf) Note Allow 0.57322 for S_{xy} and 0.76634 for S_{yy} from rounding mean of y to 0.919.
		$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{0.56595}{\sqrt{0.96826 \times 0.74796}} = 0.665$	M1 dep* A1 [5]	For structure of r For answer rounding to 0.66 or 0.67
		OR:		
		$\text{cov}(x,y) = \frac{\sum xy}{n} - \overline{x}\overline{y} = 40.66/60 - (43.62/60 \times 55.15/60)$ $= 0.0094325$	M1*	For method for cov (x,y)
		$\text{rmsd}(x) = \sqrt{\frac{S_{xx}}{n}} = \sqrt{(0.96826/60)} = \sqrt{0.016137\ldots} = 0.1270$	M1*	For method for at least one msd or rmsd
		$\text{rmsd}(y) = \sqrt{\frac{S_{yy}}{n}} = \sqrt{(0.74796/60)} = \sqrt{0.012466} = 0.1117$	A1	For at least one of cov (x,y), msd or rmsd correct (to 2 sf)
		$r = \frac{\text{cov}(x,y)}{\text{rmsd}(x)\text{rmsd}(y)} = \frac{0.0094325}{0.1270 \times 0.1117} = 0.665$	M1 dep* A1	For structure of r For answer rounding to0.66 or 0.67
			[5]	Methods mixed – max M0M1A1M0A0

(ii)		H_0: $\rho = 0$ H_1: $\rho > 0$ (one-tailed test)	B1	For H_0, H_1 in symbols. Hypotheses in words must refer to population. Do not allow alternative symbols unless clearly defined as the population correlation coefficient.	
		where ρ is the population correlation coefficient	B1	For defining ρ. Condone omission of "population" if correct notation ρ is used, but if ρ is defined as the **sample** correlation coefficient then award **B0**. Allow "ρ is the pmcc".	
		For $n = 60$, 5% critical value = 0.2144	B1	For critical value	
		Since 0.665 > 0.2144, the result is significant.	M1	For sensible comparison leading to a conclusion provided that $\lvert r \rvert < 1$. The comparison can be in the form of a diagram as long as it is clear and unambiguous. Sensible comparison: e.g. 0.665 > 0.2144 is 'sensible' whereas 0.665 > - 0.2144 is 'not sensible'. Reversed inequality sign e.g. 0.665 < 0.2144 etc. gets max M1 A0.	
		Thus we have sufficient evidence to reject H_0	A1	For reject H_0 o.e. FT their r and critical value from 5% 1-tail column.	
		There is sufficient evidence at the 5% level to **suggest** that there is **positive** correlation between FEV1 before and after the two-week course.	E1	For correct, **non-assertive** conclusion in context (allow 'x and y' for context). E0 if H_0 and H_1 not stated, reversed or mention a value other than zero for ρ in H_0.	
			[6]		

(iii)		The underlying population must have a bivariate Normal distribution. Yes, since the scatter diagram appears to have a roughly elliptical shape.	B1 E1 **[2]**	Condone “bivariate Normal distribution”, “underlying bivariate Normal distribution”, but **do not allow** “the **data** have a bivariate Normal distribution” Condone ‘oval’ or suitable diagram	
(iv)		The significance level is the probability of rejecting the null hypothesis when in fact it is true.	E1* E1dep* **[2]**	For “probability of rejecting H_0” or “probability of a significant result”. For “when H_0 is true”	
(v)		$\sum x = 43.62 + 0.45 = 44.07$ $\sum y = 55.15 - 0.45 = 54.70$ $\sum xy = 40.66$ $\sum x^2 = 32.68 + 1 - 0.55^2 = 33.3775$ $\sum y^2 = 51.44 - 1 + 0.55^2 = 50.7425$	B1 B1 B1 **[3]**	For $\sum x$ or $\sum y$ or $\sum xy$ For $\sum x^2$ or $\sum y^2$ (to 2 dp) For all correct (ignore n)	

Q8, (Jun 2014, Q1)

(i)

Diastolic	60	61	62	63	73	76	84	87	90	95
Systolic	98	121	118	114	108	112	132	130	134	139
Rank dias	1	2	3	4	5	6	7	8	9	10
Rank sys	1	6	5	4	2	3	8	7	9	10
d	0	4	2	0	-3	-3	1	-1	0	0
d^2	0	16	4	0	9	9	1	1	0	0

$\Sigma d^2 = 40$

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times40}{10\times99} = 1 - \frac{240}{990} = 1 - 0.242$$

= 0.758 (to 3 s.f.) [*allow* 0.76 to 2 s.f.]

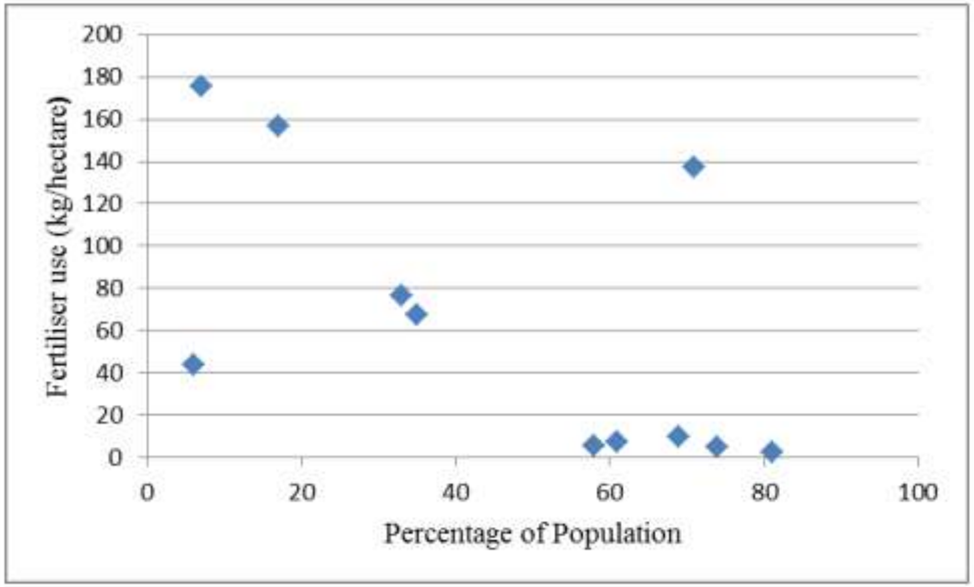
M1	For attempt at ranking (allow all ranks reversed)
M1	For d^2
A1	For 40 soi e.g. can be implied by 0.242 seen.
M1	For method for r_s using their Σd^2
A1	For 0.758 or 25/33 or 0.75 recurring f.t. their Σd^2 provided $\lvert r_s\rvert < 1$ Do not allow 0.7575 NB No ranking scores 0/5
[5]	

(ii)		H_0: no association between diastolic blood pressure and systolic blood pressure in the population of young adults	B1	
		H_1: **positive** association between diastolic blood pressure and systolic blood pressure in the population of young adults	B1	NB Hypotheses must be in context. H_0: no association, H_1: positive association, earns SC1 Hypotheses must **not** be given in terms of ρ or mention correlation. Ignore references to ρ if hypotheses also given in words.
			B1	For **population of young adults** seen at least once. Do **not** allow **underlying population**. B0 for population correlation coefficient.
		One tail test critical value at 5% level is 0.5636	B1*	for 0.5636 cv from pmcc test = 0.5494 gets B0
		Since $0.758 > 0.5636$, there is sufficient evidence to reject H_0,	M1dep*	For a sensible comparison, leading to a conclusion, of their r_s with 0.5636, provided $0 < r_s < 1$ The comparison may be in the form of a diagram.
		i.e. conclude that there is sufficient evidence to **suggest** that there is positive association between diastolic blood pressure and systolic blood pressure (in the population of young adults).	A1	For **non-assertive,** correct conclusion in context. f.t. their r_s. Allow "support" in place of "suggest". Do not allow "show", "imply", "conclude" or "prove" in place of "suggest". If a two-tailed test is carried out then award maxB1B0B1B1(for 0.6485)M0A0
			[6]	

(iv)		H_0: $\rho = 0$ H_1: $\rho > 0$	B1	Do not allow other symbols unless clearly defined as population correlation coefficient. Do not allow hypotheses solely in words.
		where ρ is the (population) correlation coefficient	B1	For defining ρ
		For $n = 10$, 1% critical value = 0.7155	B1*	For 0.7155
		Since 0.707 < 0.7155 the result is not significant	M1dep*	For sensible comparison leading to a conclusion. Conclusion soi.
		There is insufficient evidence at the 1% level to **suggest** that there is **positive** correlation between diastolic blood pressure and systolic blood pressure (in this population).	A1	For **non-assertive** correct conclusion in context. If a two-tailed test is carried out then award maxB0B1B1(for 0.7646)M0A0
			[5]	

Q9, (Jun 2016, Q1)

(i)



G1	For suitably labelled axes. Condone absence of scale here.
G2,1,0	G2 for 11 points correctly plotted relative to a suitable **linear** scale. G1 if 9 or 10 correctly plotted. G0 if 3 or more incorrectly plotted/omitted Allow axes interchanged
[3]	

(ii)

(The points in the scatter diagram) do not appear to be roughly elliptical.
The **population** may not have a bivariate Normal distribution.

E1	For "not elliptical".
E1	For not **underlying** bivariate Normal. Do not allow "the data" in place of population/underlying. Allow "data is not **from** a bivariate Normal distribution".
[2]	Do not allow Normal bivariate ….

(iii)

Percentage	33	6	58	35	81	69	61	7	74	71	17
Fertiliser use	76	44	6	68	3	10	7	176	5	137	157
Rank percentage	4	1	6	5	11	8	7	2	10	9	3
Rank Fertiliser	8	6	3	7	1	5	4	11	2	9	10
d	4	5	-3	2	-10	-3	-3	9	-8	0	7
d^2	16	25	9	4	100	9	9	81	64	0	49

M1	For ranking (allow ranks reversed) **NB No ranking scores 0/5**
M1	For d^2

		$\Sigma d^2 = 366$	A1	For Σd^2 (May be embedded in the calculation)	
		$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\times366}{11\times120} = 1 - \frac{2196}{1320} = 1 - 1.6636$	M1	For method for r_s	
		= - 0.664 (to 3 s.f.) [*allow* -0.66 to 2 s.f. or -73/110]	A1 **[5]**	FT their Σd^2 provided $-1 < r_s < 0$, and ranking used. **NB No ranking scores 0/5**	
(iv)		H_0: no association between percentage of population living in rural areas and fertiliser use (in the population of countries)	B1	For null hypothesis in context **NB** H_0 H_1 <u>not</u> ito ρ.	
		H_1: **negative** association between percentage of population living in rural areas and fertiliser use (in the population of countries)	B1	For alternative hypothesis in context. Context needed in at least one of the hypotheses.	
			B1	For **population of countries** or **underlying population**.	
		One tail test critical value at 1% level is -0.7091	B1	For ±0.7091 **No further marks from here if incorrect**.	
		Since - 0.664 > -0.7091 [or 0.664 < 0.7901] there is …	M1	For sensible comparison of their "- 0.664" with ±0.7091 **seen**, leading to conclusion, only if $-1 <$ their $r_s < 0$.	
		…insufficient evidence to reject H_0. There is insufficient evidence to suggest that there is **negative** association between percentage of population living in rural areas and fertiliser use (in the population of countries)	A1 **[6]**	for not significant, oe, and correct conclusion in context. FT their r_s with correct cv.	
(v)		It means that the probability of rejecting H_0 given that it is correct is 1% o.e.	E1 **[1]**	Allow "the probability of a false positive is 1%", "the probability of a Type I Error is 1%". Do not allow "It means that the probability rejecting H_0 when it should have been accepted is 1%"	
(vi)		None	E1 **[1]**		