Continuous Uniform Distribution (Fom Edexcel 6684)

Q1, (Jun 2005, Q2)

The continuous random variable X is uniformly distributed over the interval $[2, 6]$.

(a) Write down the probability density function $f(x)$.

(2)

Find

(b) $E(X)$,

(1)

(c) $Var(X)$,

(2)

(d) the cumulative distribution function of X, for all x,

(4)

(e) $P(2.3 < X < 3.4)$.

(2)

Q2, (Jun 2006, Q2)

The continuous random variable L represents the error, in mm, made when a machine cuts rods to a target length. The distribution of L is continuous uniform over the interval $[-4.0, 4.0]$.

Find

(a) $P(L < -2.6)$,

(1)

(b) $P(L < -3.0 \text{ or } L > 3.0)$.

(2)

A random sample of 20 rods cut by the machine was checked.

(c) Find the probability that more than half of them were within 3.0 mm of the target length.

(4)

Q4, (Jan 2007, Q5)

The continuous random variable X is uniformly distributed over the interval $\alpha < x < \beta$.

(a) Write down the probability density function of X, for all x.

(2)

(b) Given that $E(X) = 2$ and $P(X < 3) = \frac{5}{8}$ find the value of α and the value of β.

(4)

A gardener has wire cutters and a piece of wire 150 cm long which has a ring attached at one end. The gardener cuts the wire, at a randomly chosen point, into 2 pieces. The length, in cm, of the piece of wire with the ring on it is represented by the random variable X. Find

(c) $E(X)$,

(1)

(d) the standard deviation of X,

(2)

(e) the probability that the shorter piece of wire is at most 30 cm long.

(3)

Q5, (Jan 2009, Q2)

The continuous random variable X is uniformly distributed over the interval $[-2, 7]$.

(a) Write down fully the probability density function $f(x)$ of X.

(2)

(b) Sketch the probability density function $f(x)$ of X.

(2)

Find

(c) $E(X^2)$,

(3)

(d) $P(-0.2 < X < 0.6)$.

(2)

Q6, (Jun 2010, Q3)

A rectangle has a perimeter of 20 cm. The length, X cm, of one side of this rectangle is uniformly distributed between 1 cm and 7 cm.

Find the probability that the length of the longer side of the rectangle is more than 6 cm long.

(5)

Q7, (Jun 2011, Q4)

In a game, players select sticks at random from a box containing a large number of sticks of different lengths. The length, in cm, of a randomly chosen stick has a continuous uniform distribution over the interval [7, 10].

A stick is selected at random from the box.

(a) Find the probability that the stick is shorter than 9.5 cm.

(2)

To win a bag of sweets, a player must select 3 sticks and wins if the length of the longest stick is more than 9.5 cm.

(b) Find the probability of winning a bag of sweets.

(2)

To win a soft toy, a player must select 6 sticks and wins the toy if more than four of the sticks are shorter than 7.6 cm.

(c) Find the probability of winning a soft toy.

(4)

Q8, (Jun 2013(R), Q3)

The random variable X has a continuous uniform distribution on $[a, b]$ where a and b are positive numbers.

Given that $E(X) = 23$ and $Var(X) = 75$

(a) find the value of a and the value of b.

(6)

Given that $P(X > c) = 0.32$

(b) find $P(23 < X < c)$.

(2)

Q8, (Jan 2013, Q4)

The continuous random variable X is uniformly distributed over the interval $[-4, 6]$.

(a) Write down the mean of X.

(1)

(b) Find $P(X \leqslant 2.4)$

(2)

(c) Find $P(-3 < X - 5 < 3)$

(2)

The continuous random variable Y is uniformly distributed over the interval $[a, 4a]$.

(d) Use integration to show that $E(Y^2) = 7a^2$

(4)

(e) Find $Var(Y)$.

(2)

(f) Given that $P(X < \frac{8}{3}) = P(Y < \frac{8}{3})$, find the value of a.

(3)

Q9, (Jun 2013, Q4)

A continuous random variable X is uniformly distributed over the interval $[b, 4b]$ where b is a constant.

(a) Write down $E(X)$.

(1)

(b) Use integration to show that $Var(X) = \frac{3b^2}{4}$.

(3)

(c) Find $Var(3 - 2X)$.

(2)

Given that $b = 1$ find

(d) the cumulative distribution function of X, $F(x)$, for all values of x,

(2)

(e) the median of X.

(1)
