Continuous Random Variables (Mean, Variance and Median) (From OCR 4733)

Q1, (Jan 2006, Q8)

A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where n and k are positive constants.

(i) Find k in terms of n. [3]

(ii) Show that $E(X) = \dfrac{n+1}{n+2}$. [3]

It is given that $n = 3$.

(iii) Find the variance of X. [3]

(iv) One hundred observations of X are taken, and the mean of the observations is denoted by $\overline{X}$. Write down the approximate distribution of $\overline{X}$, giving the values of any parameters. [3]

(v) Write down the mean and the variance of the random variable Y with probability density function given by

$$g(y) = \begin{cases} 4\left(y + \frac{4}{5}\right)^3 & -\frac{4}{5} \leqslant y \leqslant \frac{1}{5}, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

Q2, (Jun 2007, Q7)

Two continuous random variables S and T have probability density functions as follows.

$$S: \quad f(x) = \begin{cases} \frac{1}{2} & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

$$T: \quad g(x) = \begin{cases} \frac{3}{2}x^2 & -1 \leqslant x \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) *Sketch* on the same axes the graphs of $y = f(x)$ and $y = g(x)$. [You should not use graph paper or attempt to plot points exactly.] [3]

(ii) Explain in everyday terms the difference between the two random variables. [2]

(iii) Find the value of t such that $P(T > t) = 0.2$. [5]

Q3, (Jun 2009, Q7i,ii)

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{9}x(3 - x) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the variance of X. **[5]**

(ii) Show that the probability that a single observation of X lies between 0.0 and 0.5 is $\frac{2}{27}$. **[2]**

Q4, (Jun 2010, Q8)

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} kx^{-a} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants and a is greater than 1.

(i) Show that $k = a - 1$. **[3]**

(ii) Find the variance of X in the case $a = 4$. **[5]**

(iii) It is given that $P(X < 2) = 0.9$. Find the value of a, correct to 3 significant figures. **[4]**

Q5, (Jan 2012, Q7)

(i) The continuous random variable X has the probability density function

$$f(x) = \begin{cases} \dfrac{1}{2\sqrt{x}} & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find **(a)** $E(X)$, **[3]**

(b) the median of X. **[3]**

(ii) The continuous random variable Y has the probability density function

$$g(y) = \begin{cases} \dfrac{1.5}{y^{2.5}} & y \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given that $E(Y) = 3$, show that $\text{Var}(Y)$ is not finite. **[3]**

Q6, (Jun 2013, Q5)

Two random variables S and T have probability density functions given by

$$f_S(x) = \begin{cases} \dfrac{3}{a^3}(x-a)^2 & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

$$f_T(x) = \begin{cases} c & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a and c are constants.

(i) On a single diagram sketch both probability density functions. **[3]**

(ii) Calculate the mean of S, in terms of a. **[5]**

(iii) Use your diagram to explain which of S or T has the bigger variance. (Answers obtained by calculation will score no marks.) **[2]**

Q7, (Jun 2014, Q5)

A continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{1}{2}\pi \sin(\pi x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that this is a valid probability density function. **[4]**

(ii) Sketch the curve $y = f(x)$ and write down the value of $E(X)$. **[3]**

(iii) Find the value q such that $P(X > q) = 0.75$. **[3]**

(iv) Write down an expression, including an integral, for $\text{Var}(X)$. (Do not attempt to evaluate the integral.) **[2]**

(v) A student states that "X is more likely to occur when x is close to $E(X)$." Give an improved version of this statement. **[1]**

Q8, (Jun 2015, Q3)

A continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{3}{2a^3}x^2 & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) It is given that $P(-3 \leqslant X \leqslant 3) = 0.125$. Find the value of a in this case. **[4]**

(ii) It is given instead that $\text{Var}(X) = 1.35$. Find the value of a in this case. **[5]**

(iii) Explain the relationship between x and X in this question. **[1]**

Q9, (Jun 2016, Q7)

A continuous random variable X has probability density function

$$f(x) = \begin{cases} ax^{-3} + bx^{-4} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) Explain what the letter x represents. **[1]**

It is given that $P(X > 2) = \frac{3}{16}$.

(ii) Show that $a = 1$, and find the value of b. **[7]**

(iii) Find $E(X)$. **[3]**
