Continuous Random Variables (Mean, Variance and Median) (From OCR 4733)

Q1, (Jan 2006, Q8)

(i)	$\int_0^1 x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$	M1		Integrate x^n, limits 0 and 1
	$k/(n+1) = 1$ so $k = n+1$	M1		Equate to 1 and solve for k
		A1	3	Answer $n+1$, *not* 1^{n+1}, c.w.o.
(ii)	$\int_0^1 x^{n+1}\,dx = \left[\frac{x^{n+2}}{n+2}\right]_0^1 = \frac{1}{n+2}$	M1		Integrate x^{n+1}, limits 0 and 1, not just $x.x^n$
		A1		Answer $\frac{1}{n+2}$
	$\mu = \frac{k}{n+2} = \frac{n+1}{n+2}$ **AG**	A1	3	Correctly obtain given answer
(iii)	$\int_0^1 x^5\,dx = \left[\frac{x^6}{6}\right]_0^1$ $[= \frac{1}{6}]$	M1		Integrate x^5, limits 0 and 1, allow with n
		M1		Subtract $\left(\frac{4}{5}\right)^2$
	$\sigma^2 = \frac{4}{6} - \left(\frac{4}{5}\right)^2 = \frac{2}{75}$	A1	3	Answer $\frac{2}{75}$ or a.r.t. 0.027
(iv)	$N\left(\frac{4}{5}, \frac{2}{7500}\right)$	B1		Normal stated
		B1		Mean $\frac{4}{5}$ or $\frac{n+1}{n+2}$
		B1√	3	Variance their (iii)/100, a.e.f., allow √
(v)	Same distribution, translated	M1		Can be negative translation; *or* integration, must include correct method for integral
	Mean 0	A1√		(Their mean) – $\frac{4}{5}$, c.w.d.
	Variance $\frac{2}{75}$	B1√		Variance same as their (iii), or $\frac{2}{75}$ by integration
		3		

Q2, (Jun 2007, Q7)

(i)		B1		Horizontal straight line
		B1		Positive parabola, symmetric about 0
		B1	3	Completely correct, including correct relationship between two Don't need vertical lines or horizontal lines outside range, but don't give last B1 if horizontal line continues past "±1"
(ii)	*S* is equally likely to take any value in range, *T* is more likely at extremities	B2	2	Correct statement about distributions (*not* graphs) *[Partial statement, or correct description for one only: B1]*
(iii)	$\int_t^1 \frac{3}{2}x^2\,dx = \left[\frac{x^3}{2}\right]_t^1$	M1		Integrate f(x) with limits (–1, t) or (t, 1) [recoverable if t used later]
		B1		Correct indefinite integral
	$\frac{1}{2}(1 - t^3) = 0.2$ or $\frac{1}{2}(t^3 + 1) = 0.8$	M1		Equate to 0.2, or 0.8 if [–1, t] used
	$t^3 = 0.6$	M1		Solve cubic equation to find t
	$t = 0.8434$	A1	5	Answer, in range [0.843, 0.844]

Q3, (Jun 2009, Q7i,ii)

(i)	$\frac{2}{9}\int_0^3 x^3(3-x)\,dx = \frac{2}{9}\left[\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^3$ [= 2.7] – $(1\frac{1}{2})^2$ $= \frac{9}{20}$ or **0.45**	M1 A1 B1 M1 A1	 5	Integrate x^2f(x) from 0 to 3 [*not* for μ] Correct indefinite integral Mean is 1½, soi [not recoverable later] Subtract their μ^2 Answer art 0.450
(ii)	$\frac{2}{9}\int_0^{0.5} x(3-x)\,dx = \frac{2}{9}\left[\frac{3x^2}{2}-\frac{x^3}{3}\right]_0^{0.5}$ $= \frac{2}{27}$ AG	M1 A1	 2	Integrate f(x) between 0, 0.5, must be seen somewhere Correctly obtain given answer $\frac{2}{27}$, decimals other than 0.5 not allowed, 1 more line needed (eg [ ] = ⅓)

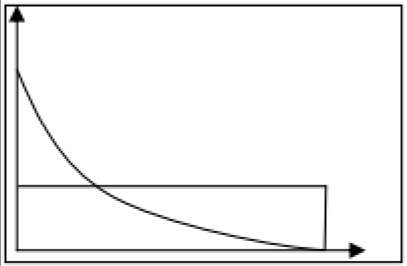
Q4, (Jun 2010, Q8)

(i)	$\int_1^\infty kx^{-a}\,dx = \left[k\frac{x^{-a+1}}{-a+1}\right]_1^\infty$ Correctly obtain $k = a - 1$ **AG**	M1 B1 A1	 3	Integrate f(x), limits 1 and ∞ (at some stage) Correct indefinite integral Correctly obtain given answer, don't need to see treatment of ∞ but mustn't be wrong. Not k^{-a+1}
(ii)	$\int_1^\infty 3x^{-3}\,dx = \left[3\frac{x^{-2}}{-2}\right]_1^\infty = 1\frac{1}{2}$ $\int_1^\infty 3x^{-2}\,dx = \left[3\frac{x^{-1}}{-1}\right]_1^\infty - (1\frac{1}{2})^2$ Answer ¾	M1 M1 A1 M1 A1	 5	Integrate xf(x), limits 1 and ∞ (at some stage) [x^4 is *not* MR] Integrate x^2f(x), correct limits Either μ = 1½ or E(X^2) = 3 stated or implied, allow k, k/2 Subtract their numerical μ^2, allow letter if subs later Final answer ¾ or 0.75 only, cwo, e.g. not from μ = –1½. [SR: Limits 0, 1: can get (i) B1, (ii) M1M1M1]
(iii)	$\int_1^2 (a-1)x^{-a}\,dx = \left[-x^{-a+1}\right]_1^2 = 0.9$ $1 - \frac{1}{2^{a-1}} = 0.9,\ 2^{a-1} = 10$ $a = 4.322$	M1* dep*M1 M1 indept A1	 4	Equate ∫f(x)dx, one limit 2, to 0.9 or 0.1. [Normal: 0 ex 4] Solve equation of this form to get 2^{a-1} = number Use logs or equivalent to solve 2^{a-1} = number Answer, a.r.t. 4.32. T&I: (M1M1) B2 or B0

Q5, (Jan 2012, Q7)

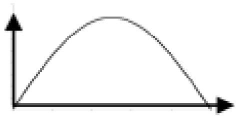
(i)	(a)	$\int_1^4 \frac{1}{2\sqrt{x}} x dx = \left[\frac{1}{3}x^{\frac{3}{2}}\right]_1^4 = 7/3$ or 2.333…	M1 B1 A1 **[3]**	Attempt to integrate $xf(x)$, correct limits Correct indefinite integral, a.e.f. Final answer 7/3 or equiv or a.r.t. 2.33
(i)	(b)	$\int_1^m \frac{1}{2\sqrt{x}} dx = 0.5$ $\sqrt{m} - 1 = 0.5$ $\boldsymbol{m = 2.25}$	M1 A1 A1 **[3]**	This or complementary integral, limits needed [not "- ∞"], equated to 0.5, needn't attempt to evaluate This equation, any equivalent simplified form Answer 9/4 or exact equivalent only
(ii)		$1.5\int_1^\infty y^{-2.5} y^2 dx = 1.5\left[\frac{y^{0.5}}{0.5}\right]_1^\infty$ Upper limit gives infinite answer	M1 B1 A1 **[3]**	Attempt to integrate $y^2f(y)$, limits 1 and ∞, allow any letter Correct indefinite integral [$=3\sqrt{y}$], ignore μ [= 3] Give correct reason, c.w.o. apart from constant, allow "= ∞"

Q6, (Jun 2013, Q5)

(i)		M1 A1 B1 **3**	Upwards parabola, not below x-axis Correct place, not extending beyond limits, ignore pointed at a Horizontal straight line, not beyond limits, y-intercept below curve (unless curve makes this meaningless)	*[scales/annotations not needed]* Touching axes (not asymptotic) Don't need vertical lines i.e., 3/3 only if wholly right
(ii)	$\int_0^a \frac{3}{a^3} x(x-a)^2 dx$ $= \int_0^a \frac{3}{a^3}(x^3 - 2ax^2 + a^2x)dx$ $= \left[\frac{3}{a^3}\left(\frac{x^4}{4} - \frac{2ax^3}{3} + \frac{a^2x^2}{2}\right)\right]_0^a$ $= \frac{a}{4}$	M1 M1 A1 B1 A1 **5**	Attempt this integral, correct limits seen somewhere Method for $\int xf(x)$, e.g. multiply out or parts, independent of first M1 Correct form for integration, e.g. multiplied out correctly, or correct first stage of parts Correct indefinite integral $\frac{a}{4}$ or exact equivalent (e.g. $0.25a$) only	 Multiplication: needs 3 terms E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \int\frac{3}{a^3}\frac{(x-a)^3}{3}dx$ E.g. $\frac{3}{a^3}x\frac{(x-a)^3}{3} - \frac{3}{a^3}\frac{(x-a)^4}{12}$ Limits not seen anywhere: can get M0M1A0B1A0

(iii)	S is concentrated more towards 0 Therefore T has bigger variance	M1 A1 **2**	Reason that shows understanding of PDF Correct conclusion	*Not*, e.g., "T is constant"

Q7, (Jun 2014, Q5)

(i)	$\int_0^1 \frac{\pi}{2}\sin(\pi x)\,dx = \left[-\frac{1}{2}\cos(\pi x)\right]_0^1 = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$ and function non-negative for all x in range	M1 B1 A1 B1 **[4]**	Attempt to integrate f(x), limits (0, 1) somewhere, evidence e.g. "from calculator" Correctly integrate $\sin(\pi x)$ to $-½\cos(\pi x)$ Fully correct, need to see $-½\cos(\pi x)$ and final 1, no wrong working seen Non-negative asserted explicitly, allow positive or equivalent. Not just graph drawn. *(Most will not get this mark!)*
(ii)	 $E(X) = ½$	M1 A1 B1 **[3]**	Correct shape, through 0, allow below axis outside range. Allow partial curve if clearly part of sine curve. Fully correct including no extension beyond [0, 1]. Don't worry about grads at ends. Ignore labelling of axes ½ or 0.5, needs to be simplified, no working needed, *no* ft
(iii)	$\int_q^1 \frac{1}{2}\pi\sin(\pi x)dx = 0.75$; $\{\left[-\frac{1}{2}\cos(\pi x)\right]_q^1 = 0.75\}$ $\cos(\pi q) = 0.5$ Solve to get $q = ⅓$	M1 A1 A1 **[3]**	Equate integral to correct probability, correct limits *somewhere* allow complementary probability (= 0.25) only if limits (0, q) $\cos(\pi q) = 0.5$ or exact equivalent $q = ⅓$ or a.r.t. 0.333. SR: Numerical (no working needed): 0.333 B3, 0.33 B2
(iv)	$\int_0^1 \frac{\pi}{2}x^2\sin(\pi x)\,dx - \left(\frac{1}{2}\right)^2$	M1 A1ft **[2]**	Integral part correct, allow limits omitted, ignore dx Subtract their $[E(X)]^2$, allow μ in form of integral, correct limits needed, not just "μ^2" {note for scoris zoning – (ii) needs to be visible here}
(v)	Values of x in range close to E(X) are more likely than those further away	B1 **[1]**	Need to see "values of x" or equivalent, and probably not "occur" *Not* "the probability of x is greater when x is close to E(X)" etc. *Not* "PDF greater …"

Q8, (Jun 2015, Q3)

(i)	$\int_{-3}^{3} \frac{3}{2a^3} x^2 dx = \left[\frac{x^3}{2a^3}\right]_{-3}^{3} = \frac{27}{a^3}$	M1dep*	Integrate, attempt at correct seen limits *somewhere*	Allow e.g. "< 3" = "≤ –4"
		B1	Correct indefinite integral, can be implied by, e.g. $27/a^3$	Allow also for a^3 on top
	= 0.125	*M1	Equate, with limits, to 0.125 and solve	
	so **$a = 6$**	A1	Solve to get $a = 6$ exactly	Allow 6.00 but no other decimals. *Not* ±6
		4		
(ii)	$\mu = 0$	B1	Stated somewhere or calculated, any a	If $\mu = 0$ not mentioned anywhere, or "– μ" stated [instead of "– μ^2"], B0 but can get remaining 4/5
	$\int_{-a}^{a} kx^4 dx = \left[k\frac{x^5}{5}\right]_{-a}^{a} = \frac{3a^2}{5}$	M1dep*	Attempt to integrate x^2f(x), limits $\pm a$	
		B1	Or exact equivalent, can be implied	
		*M1	Equate to 1.35 and solve	Don't need explicit $-\mu^2$ here
	= 1.35 so **$a = 1.5$**	A1	$a = 1.5 \pm 0.005$, allow ±1.5, ignore "must be positive"	NB: $a = 3$ is *not* MR but can get B1 for $\mu = 0$
		5		
(iii)	x is a value [values] that X takes	B1	Ignore irrelevancies or extra wrong, unless contradictory	*Not* answers just about the *function*
		1		

Q9, (Jun 2016, Q7)

(i)	[x represents a] possible value(s) taken by X	B1	**1**	Must refer to, or imply, both x and X or "the random variable" Ignore extra unless definitely wrong
(ii)	$\int_2^{\infty} ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_2^{\infty}=\frac{a}{8}+\frac{b}{24}$ *or* $\int_1^{\infty} ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_1^{\infty}=\frac{a}{2}+\frac{b}{3}$ *or* $\int_1^{2} ax^{-3}+bx^{-4}\mathrm{d}x=\left[-\frac{a}{2x^2}-\frac{b}{3x^3}\right]_1^{2}=\frac{3a}{8}+\frac{7b}{24}$ $\frac{a}{2}+\frac{b}{3}=1$ or $\frac{a}{8}+\frac{b}{24}=\frac{3}{16}$ or $\frac{3a}{8}+\frac{7b}{24}=\frac{13}{16}$ Solve to get $a=1$ $b=\frac{3}{2}$	B1 M1 M1 M1 A1 A1 B1	 **7**	Correct indefinite integral [from any set of limits or none] Integrate and substitute limits to obtain one expression Integrate and substitute limits to obtain a second expression The limits must be two of (1, ∞), (1, 2) or (2, ∞), allow (3, ∞) for "≥ 2" Equate two expressions from definite integrals to 1 or $\frac{3}{16}$ or $\frac{13}{16}$ as appropriate, and attempt to solve Both equations correct, any equivalent <u>simplified</u> form, can be implied ["simplified" = one a term, one b term, one number term] Correctly show $a = 1$ **AG**, www Correct value of b obtained from at least one correct equation SC: One equation only: M1B1 M0M0A0 A0B1, max 3/7 Two equations, assume $a = 1$, solve for b, checked in other equation: 7/7
(iii)	$\int_1^{\infty} ax^{-2}+bx^{-3}\mathrm{d}x=\left[-\frac{a}{x}-\frac{b}{2x^2}\right]_1^{\infty}$ $\left\{=a+\frac{b}{2}\right\}$ $=1\frac{3}{4}$	M1 B1ft A1	 **3**	Integrate $x\mathrm{f}(x)$, limits 1 and ∞ seen somewhere Correct indefinite integral, their b, can be implied by correct answer *Expect to see* $\int_1^{\infty} x^{-2}+\frac{3}{2}x^{-3}\mathrm{d}x=\left[-\frac{1}{x}-\frac{3}{4x^2}\right]_1^{\infty}$ Correctly obtain 1¾ or a.r.t. 1.75 www, allow from calculator