

Confidence Intervals for the Mean of a Sampling Distribution (From Edexcel 6691)

Q1, (Jun 2007, Q6)

A random sample of the daily sales (in £s) of a small company is taken and, using tables of the normal distribution, a 99% confidence interval for the mean daily sales is found to be

$$(123.5, 154.7)$$

Find a 95% confidence interval for the mean daily sales of the company.

(6)

Q2, (Jun 2008, Q1)

Some biologists were studying a large group of wading birds. A random sample of 36 were measured and the wing length, x mm of each wading bird was recorded. The results are summarised as follows

$$\sum x = 6046 \qquad \sum x^2 = 1\,016\,338$$

(a) Calculate unbiased estimates of the mean and the variance of the wing lengths of these birds.

(3)

Given that the standard deviation of the wing lengths of this particular type of bird is actually 5.1 mm,

(b) find a 99% confidence interval for the mean wing length of the birds from this group.

(5)

Q3, (Jun 2009, Q2)

The heights of a random sample of 10 imported orchids are measured. The mean height of the sample is found to be 20.1 cm. The heights of the orchids are normally distributed.

Given that the population standard deviation is 0.5 cm,

(a) estimate limits between which 95% of the heights of the orchids lie,

(3)

(b) find a 98% confidence interval for the mean height of the orchids.

(4)

A grower claims that the mean height of this type of orchid is 19.5 cm.

(c) Comment on the grower's claim. Give a reason for your answer.

(2)

Q4, (Jun 2010, Q3)

A woodwork teacher measures the width, w mm, of a board. The measured width, X mm, is normally distributed with mean w mm and standard deviation 0.5 mm.

- (a) Find the probability that X is within 0.6 mm of w . (2)

The same board is measured 16 times and the results are recorded.

- (b) Find the probability that the mean of these results is within 0.3 mm of w . (4)

Given that the mean of these 16 measurements is 35.6 mm,

- (c) find a 98% confidence interval for w . (4)

Q5, (Jun 2011, Q7)

In this question, assume that the standard deviation calculated is the unbiased estimator s^2 .

Roastie's Coffee is sold in packets with a stated weight of 250 g. A supermarket manager claims that the mean weight of the packets is less than the stated weight. She weighs a random sample of 90 packets from their stock and finds that their weights have a mean of 248 g and a standard deviation of 5.4 g.

- (a) Using a 5% level of significance, test whether or not the manager's claim is justified. State your hypotheses clearly. (5)

- (b) Find the 98% confidence interval for the mean weight of a packet of coffee in the supermarket's stock. (4)

- (c) State, with a reason, the action you would recommend the manager to take over the weight of a packet of Roastie's Coffee. (2)

Roastie's Coffee company increase the mean weight of their packets to μ g and reduce the standard deviation to 3 g. The manager takes a sample of size n from these new packets. She uses the sample mean \bar{X} as an estimator of μ .

- (d) Find the minimum value of n such that $P(|\bar{X} - \mu| < 1) \geq 0.98$ (5)
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Q6, (Jun 2013, Q5)

A manufacturer produces circular discs with diameter D mm, such that $D \sim N(\mu, \sigma^2)$. A random sample of discs is taken and, using tables of the normal distribution, a 90% confidence interval for μ is found to be

$$(118.8, 121.2)$$

(a) Find a 98% confidence interval for μ . (6)

(b) Hence write down a 98% confidence interval for the circumference of the discs. (1)

Using three different random samples, three 98% confidence intervals for μ are to be found.

(c) Calculate the probability that all the intervals will contain μ . (2)

Q7, (Jun 2015, Q4)

The weights of bags of rice, X kg, have a normal distribution with unknown mean μ kg and known standard deviation σ kg. A random sample of 100 bags of rice gave a 90% confidence interval for μ of (0.4633, 0.5127).

(a) Without carrying out any further calculations, use this confidence interval to test whether or not $\mu = 0.5$

State your hypotheses clearly and write down the significance level you have used. (3)

A second random sample, of 150 of these bags of rice, had a mean weight of 0.479 kg.

(b) Calculate a 95% confidence interval for μ based on this second sample. (6)

Q8, (Jun 2016, Q7)

In this question, assume that the standard deviation calculated is the unbiased estimator s^2 .

A restaurant states that its hamburgers contain 20% fat. Paul claims that the mean fat content of their hamburgers is less than 20%. Paul takes a random sample of 50 hamburgers from the restaurant and finds that they contain a mean fat content of 19.5% with a standard deviation of 1.5%

You may assume that the fat content of hamburgers is normally distributed.

- (a) Find the 90% confidence interval for the mean fat content of hamburgers from the restaurant. (4)

- (b) State, with a reason, what action Paul should recommend the restaurant takes over the stated fat content of their hamburgers. (2)

The restaurant changes the mean fat content of their hamburgers to $\mu\%$ and adjusts the standard deviation to 2%. Paul takes a sample of size n from this new batch of hamburgers. He uses the sample mean \bar{X} as an estimator of μ .

- (c) Find the minimum value of n such that $P(|\bar{X} - \mu| < 0.5) \geq 0.9$ (5)

Q9, (Jun 2017, Q5)

Paul takes the company bus to work. According to the bus timetable he should arrive at work at 0831. Paul believes the bus is not reliable and often arrives late. Paul decides to test the arrival time of the bus and carries out a survey. He records the values of the random variable

$$X = \text{number of minutes after 0831 when the bus arrives.}$$

His results are summarised below.

$$n = 15 \quad \sum x = 60 \quad \sum x^2 = 1946$$

- (a) Calculate unbiased estimates of the mean, μ , and the variance of X . (3)

Using the mean of Paul's sample and given $X \sim N(\mu, 10^2)$

- (b) (i) calculate a 95% confidence interval for the mean arrival time at work for this company bus.

- (ii) State an assumption you made about the values in the sample obtained by Paul. (5)

- (c) Comment on Paul's belief. Justify your answer. (2)