Confidence Intervals for the Mean of a Sampling Distribution (From Edexcel 6691)

Q1, (Jun 2007, Q6)

$\bar{x} = \frac{1}{2}(123.5+154.7) = 139.1$		B1
	2.5758	B1
"their 2.5758" $\frac{\sigma}{\sqrt{n}} = 154.7 - 139.1 = 15.6$		M1
	AWRT 1.96	B1
"their 1.96" $\frac{\sigma}{\sqrt{n}} = \frac{15.6 \times 1.96}{2.5758} = (11.87...)$		M1
So 95% C.I. = 139.1 $\pm$ 11.87… = (127.22…, 150.97…)	AWRT (127, 151)	A1
		6

Q2, (Jun 2008, Q1)

(a)	$\bar{x} = \left(\frac{6046}{36} =\right) 167.94...$	awrt **168**	B1
	$s^2 = \frac{1016338 - 36 \times \bar{x}^2}{35}$		M1
	$= 27.0253....$	awrt **27.0** (Accept 27)	A1 (3)
(b)	99% Confidence Interval is: $\bar{x} \pm 2.5758 \times \frac{5.1}{\sqrt{36}}$		M1A1ft
		2.5758	B1
	= (165.755…, 170.133…)	awrt (**166,170**)	A1 A1 (5)
			8 marks

Q3, (Jun 2009, Q2)

(a)	Limits are $20.1 \pm 1.96 \times 0.5$	M1 B1
	(19.1, 21.1)	A1cso (3)
(b)	98 % confidence limits are	
	$20.1 \pm 2.3263 \times \frac{0.5}{\sqrt{10}}$	M1 B1
	(19.7, 20.5)	A1A1 (4)
(c)	The growers claim is not correct Since 19.5 does not lie in the interval (19.7, 20.5)	B1 dB1 (2) **[9]**

Q4, (Jun 2010, Q3)

(a) $E \sim \mathrm{N}(0, 0.5^2)$ or $X \sim \mathrm{N}(w, 0.5^2)$

$\mathrm{P}(|E| < 0.6) = \mathrm{P}\left(|Z| < \frac{0.6}{0.5}\right)$ or $\mathrm{P}(|X - w| < 0.6) = \mathrm{P}\left(|Z| < \frac{0.6}{0.5}\right)$ M1

$= \mathrm{P}(|Z| < 1.2)$

$= 2 \times 0.8849 - 1 = 0.7698$ awrt **0.770** A1

(2)

(b) $\bar{E} \sim \mathrm{N}\left(0, \frac{1}{64}\right)$ or $\bar{X} \sim \mathrm{N}\left(w, \frac{0.5^2}{16}\right)$ M1

$\mathrm{P}(|\bar{E}| < 0.3) = \mathrm{P}\left(|Z| < \frac{0.3}{\frac{1}{8}}\right)$ or $\mathrm{P}(|\bar{X} - w| < 0.3) = \mathrm{P}\left(|Z| < \frac{0.3}{\frac{1}{8}}\right)$ M1, A1

$= \mathrm{P}(|Z| < 2.4)$

$= 2 \times 0.9918 - 1 = 0.9836$ awrt **0.984** A1

(4)

(c) $35.6 \pm 2.3263 \times \frac{1}{8}$ M1 B1

(35.3, 35.9) A1,A1

(4)

10

Q5, (Jun 2011, Q7)

(a)	$H_0: \mu = 250$, $H_1: \mu < 250$,	B1
	$z = \dfrac{248 - 250}{\dfrac{5.4}{\sqrt{90}}}$	M1
	$= -3.513...$ awrt -3.51	A1
	Critical value -1.6449	B1
	-3.513..<-1.6449 so sufficient evidence to reject H_0	
	Manager's claim is justified.	A1
		(5)
(b)	98% CI for μ is	
	$248 \pm 2.3263 \times \dfrac{5.4}{\sqrt{90}}$	M1B1
	= awrt (247,249) dependent upon z value awrt 2.33	A1A1
		(4)
(c)	Hypothesis test is significant or CI does not contain stated weight.	B1
	(Manager should ask the company to investigate if their) stated weight is too high o.e.	B1
		(2)
(d)	$P(\lvert\bar{x} - \mu\rvert < 1) = 0.98$	
	$\dfrac{1}{\dfrac{3}{\sqrt{n}}} = 2.3263$	M1 A1
	$n = (3 \times 2.3263)^2 = 48.7...$	dM1A1
	Sample size 49 required.	A1
		(5)
		16

Q6, (Jun 2013, Q5)

(a)	$\bar{x} = \frac{1}{2}(118.8 + 121.2) = 120$	B1
	1.6449 (or better)	B1
	" their 1.6449" $\frac{\sigma}{\sqrt{n}} = 121.2 - 120$	M1
	2.3263 (or better)	B1
	" their 2.3263" $\frac{\sigma}{\sqrt{n}} = 2.3263 \times \left(\frac{121.2 - 120}{1.6449}\right)$	dM1
	So 98% C.I. = $120 \pm 1.424\ldots = (118.3028\ldots, 121.699\ldots)$	
	awrt **(118, 122)**	A1
		(6)
(b)	awrt $(118\pi, 122\pi)$ or (371/372, 382/383)	B1ft
		(1)
(c)	P (All) $= (0.98)^3$	M1
	$= 0.941$	A1
		(2)
		[Total 9]

Q7, (Jun 2015, Q4)

(a)	$H_0 : \mu = 0.5$ $H_1 : \mu \neq 0.5$	B1
	(Significance level =)10%	dB1
	(0.5 is in the interval so not significant, accept H_0, can accept) $\mu = 0.5$	B1
		(3)
(b)	$1.6449 \times \frac{\sigma}{\sqrt{100}} = 0.0247$	M1
		B1
	$\sigma = 0.15016$ or $\frac{10 \times 0.0247}{1.6449}$ (awrt 0.15)	A1
	$0.479 \pm 1.96 \times \frac{"\sigma"}{\sqrt{150}}$	M1
		B1
	awrt **(0.455, 0.503)**	A1
		(6)
		Total 9

Q8, (Jun 2016, Q7)

(a)	$19.5 \pm 1.6449 \times \frac{1.5}{\sqrt{50}}$		M1B1
	$= (19.151..., 19.848...)$	awrt 19.2, awrt 19.8	A1A1
			(4)
(b)	CI does not contain 20 oe		M1
	Fast Food restaurant statement is too high; they should reduce the stated value.		A1
			(2)
(c)	$P(\lvert \bar{X} - \mu \rvert < 0.5) = 0.9$		
	$\frac{0.5}{\frac{2}{\sqrt{n}}} = 1.6449$		M1A1
	$n = \left(2 \times \frac{1.6449}{0.5} \right)^2 = 43.29...$		dM1A1
	Sample size required is 44		A1
			(5)
			Total 11

Q9, (Jun 2017, Q5)

(a)	$\bar{x} = \frac{60}{15} = 4$	4 cao	B1
	$s^2 = \frac{1}{14}\left(1946 - 15\times 4^2\right) = 121.857...$	M1 Use of complete, correct formula and attempt to substitute. A1 awrt 122 or $\frac{853}{7}$	M1,A1
			(3)
(b)(i)	$\bar{x} \pm 1.96\times\frac{10}{\sqrt{15}} = 4 \pm 5.06$	Accept use of $\bar{x} \pm z\times\frac{10 \text{ or "their } s\text{"}}{\sqrt{15}}$, A1 all correct. Accept $\bar{x} = 0835$.	M1,A1
	$(-1.06, 9.06)$	Can be implied from correct interval below.	A1
	$(08\,29.56, 08\,40.04)$	Accept $(08\,29.94, 0840.06)$ or expressed using words or as an inequality. Accept answers to the nearest minute ie (0830,0840).	A1
(ii)	Paul samples times of **buses randomly** or **independently** of each other	Context required.	B1
			(5)
(c)	0 / 0831 / 8.31(am) is 'contained in' the confidence interval	Award if comment about their interval is correct. Only accept 'above the lower limit of' etc if the statement taken as a whole clearly means 'contained in'.	M1
	Paul's belief is not supported / 0831 arrival time is reasonable	Must contain some context	A1cao
			(2)
			Total 10