Motion in a Vertical Circle (From OCR 4730)

Q1, (Jan 2008, Q6)

(i)	[½m7² = ½mv² + 2mg]	M1		For using the principle of conservation of energy
	Speed is 3.13ms^{-1}	A1		
	[T = mv²/r]	M1		For using Newton's second law horizontally and a = v²/r
	Tension is 1.96N	A1ft	4	
(ii)	[T – mgcosθ = mv²/r]	M1		For using Newton's second law radially
		M1		For using T = 0 (may be implied)
	v² = -2gcosθ	A1		
		M1		For using the principle of conservation of energy
	½m7² = ½mv² +mg(2 – 2cosθ)	A1		
	[-2gcosθ = 49 –4g + 4gcosθ]	M1		For eliminating v²
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	

Alternative for candidates who eliminate v² before using T = 0.

(ii)	[T – mgcosθ = mv²/r]	M1		For using Newton's second law radially
		M1		For using the principle of conservation of energy
	½m7² = ½mv² +mg(2 – 2cosθ)	A1		
	[T - mgcosθ = m(49 –4g + 4gcosθ)2]	M1		For eliminating v²
		M1		For using T = 0 (may be implied)
	-2g cosθ = 49 –4g + 4gcosθ	A1ft		ft error in energy equation
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	

Q2, (Jun 2009, Q7)

i	Gain in PE = $mga(1 - \cos\theta)$	B1	
	$[\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mga(1 - \cos\theta)]$	M1	For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos\theta)$	A1	
	$[R - mg\cos\theta = m(\text{accel.})]$		
	$R = mv^2/a + mg\cos\theta$	M1	For using Newton's second law radially
		A1	
	$[R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta]$	M1	For substituting for v^2
	$R = mu^2/a + mg(3\cos\theta - 2)$	A1	AG
		[7]	
ii	$[0 = mu^2/a - 5mg]$	M1	For substituting $R = 0$ and $\theta = 180^\circ$
	$u^2 = 5ag$	A1	
			For substituting for u^2 (= $5ag$) and θ = 180° in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2}
	$[v^2 = 5ag - 4ag]$	M1	
	Least value of v^2 is ag	A1	AG
		[4]	
iii	$[0 = u^2 - 2ga(1 - \sqrt{3}/2)]$	M1	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i))
	$u^2 = ag(2 - \sqrt{3})$	A1	Accept $u^2 = 2ag(1 - \cos\pi/6)$
		[2]	

Q3, (Jan 2012, Q7)

(i)		Tension in string $T = mg\sin\alpha$ For using $e = R\alpha - 2R/3$ $1.8\alpha - \sin\alpha - 1.2 = 0$ Finding f(1.175) and f(1.185) correctly correct conclusion	M1 B1 B1 A1 M1 A1 A1 **[7]**	For using $T = \lambda x/L$ $mg\sin\alpha = 1.2mg\left(Ra - \frac{2R}{3}\right) \div \frac{2R}{3}$ AG establish result ≈ -0.008, and $\approx +0.0065$ AG $\alpha = 1.18$ correct to 3 significant figures	By iteration $\alpha = (1.2 + \sin\alpha)/1.8$ M1 start [1, 2], and 1 iteration A1 at least 1 more iteration, and conclusion 1.18(0427) A1
(ii)		Direction is towards O	B1 **[1]**		
(iii)		 Gain in EE = $1.2mg(1.18R - 2R/3)^2 \div (2\text{x}2R/3)$ PE loss = $mgR(\cos 2/3 - \cos 1.18)$ $v^2 =$ $2gR[\cos 2/3 - \cos 1.18 - 0.9(1.18 - 2/3)^2]$ Acceleration is 3.29 ms^{-2}.	M1* A1 A1 M1 A1 *M1 A1 **[7]**	For using EE = $\lambda e^2 \div (2L)$ and PE = mgh ignore signs For using ½ mv^2 = PE loss – EE gain For using acceleration = v^2/R	 allow α for 1.18 for A1A1 allow sign errors need 1.18 here If candidates use $mR\ddot{\theta}$ use equivalent scheme

Q4, (Jun 2012, Q4)

(i)		M1	For using the pce condone sin/cos and sign errors; need KE before and after and difference in PE
	$\frac{1}{2}mv^2 + mg(0.6)(1-\cos\theta) = \frac{1}{2}m4^2$	A1	
	$v^2 = 4.24 + 11.76\cos\theta$	A1	AG
		M1	For using Newton's 2nd law, condone sin/cos and sign erorrs; 3 terms needed
	$R - 0.45g\cos\theta = 0.45v^2/0.6$	A1	
	$R = 3.18 + 13.23\cos\theta$	A1	
		[6]	
(ii)		M1	For using $R = 0$
	$\cos\theta = -3.18/13.23$	A1 FT	$-0.24036\ldots$ or $-106/441$ or $\theta = 103.9°$ ft from $R = A + B\cos\theta$, where $A, B \neq 0$
	$[v^2 = 4.24 - 11.76 \times 3.18/13.23]$	M1	For substituting for $\cos\theta$
	Speed is 1.19 ms^{-1}	A1	CAO without wrong working
		[4]	

Q5, (Jun 2015, Q7)

(i)	using $F = ma$	M1	must have the right 3 terms; allow sign error / sin for cos for M1	
	$T - 0.2g\cos\theta = 0.2v^2/0.5$	A1		
	by energy	M1		
	$\frac{1}{2}\times 0.2u^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$	A1		$v^2 = u^2 - 9.8(1 - \cos\theta)$
	$T = 5.88\cos\theta + 0.4u^2 - 3.92$	A1 **[5]**	**AG** with no errors and no gaps in argument	
(ii)	when $\theta = 180°$, $5.88\cos\theta + 0.4u^2 - 3.92 = 0$	M1	allow inequalities for M1A1	
	$-5.88 + 0.4u^2 - 3.92 = 0$	A1		
	min u is 4.95 (m s^{-1})	A1 **[3]**	$\frac{7}{2}\sqrt{2}$	4.9497... Not > 4.95
	OR, at top, $mg = \frac{mv^2}{r}$, so $v^2 = 0.5g$	B1	allow inequalities for B1M1	
	by energy $\frac{1}{2}0.2u^2 = \frac{1}{2}0.2 \times 0.5g + 0.2g$	M1		
	min u is 4.95 (m s^{-1})	A1		
(iii)	$5.88\cos\theta + 0.4\times 12.25 - 3.92 = 0$	M1		
	$\cos\theta = (3.92 - 4.9)/5.88$ $(= -1/6)$	A1	might see $\theta = 99.6°$ or 1.74 radians	99.49406...°, 1.73824...rads
	use energy eq^n from **(i)**	M1	accept use of their θ	
	$\frac{1}{2}\times 0.2\times 3.5^2 = \frac{1}{2}\times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$			
	$v = 0.904$ m s^{-1}	A1 **[4]**	$\frac{7}{30}\sqrt{15}$	0.903696...
	OR use T equation from (i)	M1		
	$0 - 0.2g(-1/6) = 0.2v^2/0.5$			
	$v = 0.904$ m s^{-1}	A1		

Q6, (Jun 2016, Q5)

(i)	By energy $\frac{1}{2}m(4ag) = \frac{1}{2}mv^2 + mga(1+\cos\theta)$ Use of $F = ma$ $T + mg\cos\theta = \frac{mv^2}{a}$ $T = 2mg - 3mg\cos\theta$ Slack when $\cos\theta = \frac{2}{3}$ Height is $\frac{5}{3}a$	M1 A1 M1 A1 M1 A1 **6**	Need correct 3 terms; allow wrong sign, missing/extra g, missing m / a; sin for cos correct Need 3 terms and g, allow sign slip, sin for cos Ft if their T has right form	$v^2 = 2ag - 2ag\cos\theta$
(ii)	If $\theta > \pi/2$, $\frac{1}{2}mU^2 > mga$ $U > \sqrt{(2ag)}$ For no complete revolutions $\frac{1}{2}mU^2 < \frac{1}{2}mu^2 + 2mga$ and $mg = m\frac{u^2}{a}$ $U < \sqrt{(5ag)}$ OR Use $\frac{1}{2}mU^2 = \frac{1}{2}mv^2 + mga(1+\cos\theta)$ and $T + mg\cos\theta = \frac{mv^2}{a}$ To get $T = m\frac{U^2}{a} - 2mg - 3mg\cos\theta$ oe When T = 0, $U^2 = 2ag + 3ag\cos\theta$ $(\theta = 0)$ gives $U < \sqrt{(5ag)}$ $(\theta = \frac{\pi}{2})$ gives $U > \sqrt{(2ag)}$	M1 A1 M1 M1 A1 **5** (M1) (A1) (M1) (A1) (A1)	Allow '=' for all M marks Allow ≥ Allow wrong sign Allow wrong sign Allow ≤ Allow ≤,< Allow ≤ Allow ≥	u is vel at top $\sqrt{2ag} < U < \sqrt{5ag}$