Linear Motion With Variable Acceleration MS (From OCR 4730)

Q1, (Jan 2007, Q5)

(i) $[mg - mkv^2 = ma]$	M1		For using Newton's second law
$(v\,dv/dx)/(g - kv^2) = 1$	A1	2	AG
(ii) $[-\frac{1}{2}[\ln(g - kv^2)]/k = x \quad (+C)]$	M1		For separating variables and attempting to integrate
$[-(\ln g)/2k = C]$	M1		For using v(0) = 0 to find C
$x = [-\frac{1}{2}[\ln\{(g - kv^2)/g\}]/k$	A1		Any equivalent expression for x
$[\ln\{(g - kv^2)/g\} = \ln(e^{-2kx})]$	M1		For expressing in the form $\ln f(v^2) = \ln g(x)$ or equivalent
$v^2 = (1 - e^{-2kx})g/k$	A1		
	M1		For using $e^{-Ax} \rightarrow 0$ for +ve A
Limiting value is $\sqrt{g/k}$	A1ft	7	AG
(iii) $[1 - e^{-600k} = 0.81]$	M1		For using $v^2(300) = 0.9^2 g/k$
$[-600k = \ln(0.19)]$	M1		For using logarithms to solve for k
$k = 0.00277$	A1	3	

Q2, (Jun 2007, Q3)

(i) $[0.2v\,dv/dx = -0.4v^2]$	M1		For using Newton's second law with a = v dv/dx
$(1/v)\,dv/dx = -2$	A1	2	AG
(ii) $[\int (1/v)dv = \int -2dx]$	M1		For separating variables and attempting to integrate
$\ln v = -2x \quad (+C)$	A1		
$[\ln v = -2x + \ln u]$	M1		For using v(0) = u
$v = ue^{-2x}$	A1	4	AG
(iii) $[\int e^{2x}dx = \int u dt]$	M1		For using v = dx/dt and separating variables
$e^{2x}/2 = ut \quad (+C)$	A1		
$[e^{2x}/2 = ut + \frac{1}{2}]$	M1		For using x(0) = 0
$u = 6.70$	A1	4	Accept $(e^4 - 1)/8$

ALTERNATIVE METHOD FOR PART (iii)

$[\int \frac{1}{v^2}dv = -2\int dt \rightarrow -1/v = -2t + A$, and $A = -1/u]$	M1		For using a = dv/dt, separating variables, attempting to integrate and using v(0) = u
	M1		For substituting $v = ue^{-2x}$
$-e^{2x}/u = -2t - 1/u$	A1		
$u = 6.70$	A1	4	Accept $(e^4 - 1)/8$

Q3, (Jan 2009, Q6)

(i)	Initial speed in medium is $\sqrt{2g \times 10}$ $(= 14)$	B1	
	$[0.125dv/dt = 0.125g - 0.025v]$	M1	For using Newton's second law with $a = dv/dt$ (3 terms required)
	$\int \frac{5dv}{5g - v} = \int dt$	M1	For separating variables and attempt to integrate
	$-5 \ln(5g - v) = t (+A)$	A1	
	$[-5 \ln 35 = A]$	M1	For using $v(0) = 14$
	$t = 5 \ln\{35/(49 - v)\}$	A1	
		M1	For method of transposition
	$v = 49 - 35e^{-0.2t}$	A1	AG
		[8]	
(ii)		M1	For integrating to find x(t)
	$x = 49t + 175e^{-0.2t} (+B)$	A1	
	$[x(3) = (49x3 + 175e^{-0.6}) - (0 + 175)]$	M1	For using limits 0 to 3 or for using $x(0) = 0$ and evaluating x(3)
	Distance is 68.0m	A1	
		[4]	

Q4, (Jun 2009, Q4)

i		M1	For using Newton's second law with $a = v(dv/dx)$
	$F - 0.25v^2 = 120v(dv/dx)$	A1	
	$F = 8000/v$	B1	
	$[32000 - v^3 = 480v^2(dv/dx)]$	M1	For substituting for F and multiplying throughout by $4v$ (or equivalent)
	$\frac{480v^2}{v^3 - 32000}\frac{dv}{dx} = -1$	A1	AG
		[5]	
ii	$\int \frac{480v^2}{v^3 - 32000} dv = -\int dx$	M1	For separating variables and integrating
	$160 \ln(v^3 - 32000) = -x \quad (+A)$	A1	
	$160 \ln(v^3 - 32000) = -x + 160 \ln 32000$ or $160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$	M1	For using $v(0) = 40$ or $[160 \ln(v^3 - 32000)]^{v}_{40} = [-x]^{500}_{0}$
		A1ft	ft where factor 160 is incorrect but +ve,
	$(v^3 - 32000)/32000 = e^{-x/160}$	B1ft	Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
	Speed of m/c is $32.2 ms^{-1}$	B1	
		[6]	

Q5, (Jun 2011, Q3)

i	$0.25(dv/dt) = -0.2v^2$	M1	For using Newton's second law with $a = dv/dt$. Allow sign error and/or omitting mass
	$0.25\int v^{-2}dv = -0.2t(+C)$	dep M1	For separating variables and attempting to integrate (ie get v^{-1} and t).
	$-v^{-1}/4 = -t/5 + C$	A1	
	$[1/4v = t/5 + 1/20]$	M1	For using $v(0) = 5$ to obtain C
	$v = \dfrac{5}{4t+1}$ oe	A1 [5]	
ii		M1	For using $v = dx/dt$ and integrating
	$x = (5/4)\ln(4t + 1)$ $(+ B)$	A1	
	Subst $v = 0.2$ in (i) to find t	M1	Implied by $t = 6$
	Obtain $x(6)$ (= 1.25 ln25 oe (4.02359...))	M1	
	Average speed is 0.671 ms^{-1}	A1 [5]	May be written as $\frac{5}{12}\ln 5$
	Alternatively		
		M1	For using $mv(dv/dx) = -0.2v^2$, separating variables and integrating. Allow sign error and/or omitting mass.
	$\ln v = -0.8x + B$	A1	
	Subst $v = 0.2$ in (i) to find t	M1	Implied by $t = 6$
	Obtain $x(0.2)$ (= 1.25 ln(5/0.2) oe (4.0239...))	M1	
	Average speed is 0.671 ms^{-1}	A1	May be written as $\frac{5}{12}\ln 5$

Q6, (Jan 2013, Q3)

(i)	Use of $F = ma$, using $v\dfrac{dv}{dx}$	M1*		Allow sign error / 0.3 omitted
	$0.3v\dfrac{dv}{dx} = 1.5x$	A1		
	Attempt to rearrange and integrate	*M1	$0.3v^2 = 1.5x^2(+c)$	No need for c. At least one side integrated correctly
	$v = \sqrt{5}x$ **AG**	A1 [4]	correct derivation WWW	
(ii)	Integrate to find x in terms of t	M1	$dx/x = \sqrt{5}dt$ and int 1 side correctly	Need to separate variables
	$\ln x = \sqrt{5}t + c$	A1		No need for c for first 2 marks
	$x = e^{\sqrt{5}t}$	A1		Must include showing c = 0.
	$v = \sqrt{5}\, e^{\sqrt{5}t}$	A1 [4]	CAO	
	OR Integrate to find v in terms of t	M1	Use jn $0.3\dfrac{dv}{dt} = 1.5x$ and int 1 side correctly	No need for c for first 2 marks
	$\dfrac{dv}{v} = \sqrt{5}dt$			
	$\ln v = \sqrt{5}t + c$	A1		
	$\ln v = \sqrt{5}t + \ln(\sqrt{5})$	A1		Must include showing c = ln(√5)
	$v = \sqrt{5}\, e^{\sqrt{5}t}$	A1	CAO	

Q7, (Jun 2014, Q4)

(i)	Use $F = mv\frac{dv}{dx}$ $-4v = \frac{dv}{dx}$ $-4x = \ln v + c$ $0 = \ln 2 + c$ $\ln\frac{v}{2} = -4x$ $v = 2e^{-4x}$	M1 A1 M1 M1 A1 **[5]**	 expression for $\frac{dv}{dx}$ required get (+/–) $Ax = \ln v + c$ valid attempt to find c need a step leading to given answer AG	Allow sign error, missing m or g inc
(ii)	$e^{4x}\,dx = 2\,dt$ $\frac{1}{4}e^{4x} = 2t + c$ $\frac{1}{4} = 0 + c$ $e^{4x} = 4(1 + \frac{1}{4})$ $x = \frac{1}{4}\ln 5$	M1* A1 *M1 *M1 A1 **[5]**	Write v as $\frac{dx}{dt}$ and separate variables must have c or use limits valid attempt to find c or subst limits find x when $t = 0.5$ - need to remove exp; allow even if no c Accept 0.402(359…)	$dv/4v^2 = -dt$ $\frac{1}{v} = 4t + \frac{1}{2}$ $\frac{dx}{dt} = \frac{2}{8t+1}$ OR $t = 0.5$ gives $v = 0.4$ $x = \frac{1}{4}\ln(8t + 1) + c$ OR $-4x = \ln 0.2$ $x = \frac{1}{4}\ln 5$
(i)	Take moments about A for whole body Wx2Lcos60° + 2Wx6Lcos60° = Rx8Lcos60° $R = 1.75W$ $S = 1.25W$	M1 A1 A1 B1 **[4]**	Correct 3 terms needed; dim correct cos60° may be omitted at least 1 correct step to show given answer	Allow sign errors, $W/2W$, cos/sin, R is reaction at C S is reaction at A For less efficient methods, M1 can only be earned when equation with one unknown, R, is reached.

Q8, (Jun 2015, Q4)

(i)	$-\frac{v}{8} = 0.4\frac{dv}{dt}$	M1*	allow sign error, allow $0.4a$
		A1	
	$t = -3.2\int\frac{1}{v}dv$	*M1	attempt to separate variables and integrate
	$t = -3.2\ln v + 3.2\ln 10$	A1	
	time taken = 3.2ln2 or 2.22 (s)	A1 **[5]**	
(ii)	$-\frac{v}{8} = 0.4v\frac{dv}{dx}$	M1*	allow sign error
		A1	
	$x = -3.2\int dv$	*M1	attempt to separate variables and integrate
	$x = -3.2\,v + 32$	A1	$x = 16$ when $v = 5$.
	ave speed = x/(i)	*M1	
	ave speed = 7.21	A1 **[6]**	
	OR		
	$\frac{dx}{dt} = 10e^{-\frac{t}{3.2}}$	M1*	for M1, ft from (i), must contain ln term
		A1	
	$x = 10\int e^{-\frac{t}{3.2}}\,dt$	*M1	attempt to separate variables and integrate
	$x = 32\left(1 - e^{-\frac{t}{3.2}}\right)$	A1	must show constant or use limits correctly
	ave speed = x/(i)	*M1	
	ave speed = 7.21	A1	dep all 5 previous marks

Q9, (Jun 2016, Q2)

(i)	$3\cos 2t = 0.2\dfrac{dv}{dt}$	M1*	Use of F = ma; condone wrong / missing 0.2 and wrong sign
	$7.5\sin 2t = v\ (+c)$ oe	*M1	Attempt to integrate, one side correct; condone missing c,
	$v = 7.5\sin 2t + 4$	A1	CAO
	11.5 (ms^{-1}) and −3.5 (ms^{-1})	A1 **4**	Depends on both M marks and fully correct working
(ii)	$x = -\frac{15}{4}\cos 2t + 4t\ (+c)$	M1*	Ft if (i) has sin or cos term
	Ave speed is their distance/$\frac{\pi}{2}$	*M1	
	$\left(\frac{15}{4} + 6\pi\right) - \left(-\frac{15}{4} + 4\pi\right)$	*M1	$\{x(\frac{3}{2}\pi) - x(\pi)\}$ found;
	8.77 (ms^{-1})	A1 **4**	CAO Accept $\frac{15}{\pi} + 4$