Coefficient of Restitution MS (From OCR 4729)

Q1, (Jan 2007, Q2)

$e = 1 = (y-x)/4$	B1		or $\frac{1}{2}x0.2x^2 + \frac{1}{2}x0.1y^2 =$	
$0.8 = 0.2x + 0.1y$	B1		$\frac{1}{2}x0.2x4^2$ (B1/B1 for any 2)	
solving sim. equ.	M1		not if poor quad. soln.	
$x = 4/3$ only	A1	4		**4**

Q2, (Jan 2007, Q3)

(i)	$x^2 = 21^2 + 2x40x9.8$	M1			
	$x = 35$	A1			
	$0 = y^2 - 2x40x9.8$	M1			
	$y = 28$	A1		may be implied	
	$e = 28/35$	M1			
	$e = 0.8$	A1	6	aef	
(ii)	$0.2x28 - -0.2x35$	M1		must be double negative	
	$I = 12.6$	A1	2		**8**

Q3, (Jun 2005, Q4)

(i)	$5m = mu + 4m$	M1		cons. of mom.	
	$u = 1$	A1			
	$e = (2-1)/5$	M1			
	$e = 0.2$	A1	4		
(ii)	$I = 4m$	B1			
	$\rightarrow$	B1	2	to the right	
(iii)	$4m = 5mv$	M1			
	$v = 0.8$	A1			
	< 1	B1	3		**9**

Q4, (Jun 2006, Q8)

(i)	$10 = 4 + m.x$	M1		conservation of momentum	
	$e = \ldots$ or rationale for $x = 2$	M1			
	$m = 3$	A1	3		
(ii)	$v = 6$	B1			
	$e = 4/5$ or 0.8	M1		allow sign errors for M mark	
		A1	3	watch out for lost minuses	
(iii)	$10 - 5 = 2x + y$ $(5 = -2a + b)$ $(-5 = 2c + d)$	M1			
		A1			
	$e = 0.8 = (y-x)/10$	M1		look for consistency	
	$y = x + 8$ $(a + b = 8)$ $(c - d = 8)$	A1			
	$x = -1$ $(a=1)$ $(c=1)$	A1		or 1 in opp. direction to 1st	
	$y = 7$ $(b=7)$ $(d=-7)$	A1			
	$\frac{1}{2}.2.5^2 + \frac{1}{2}.1.5^2 - \frac{1}{2}.2.1^2 - \frac{1}{2}.1.7^2$	M1		K.E. lost. Must be 4 parts	
	12 J	A1	8	(37.5 – 25.5)	**14**

Q5, (Jun 2009, Q6)

(i)	$I = 0.9 = 6\times0.2 - v \times 0.2$	M1		needs to be mass 0.2
		A1		
	$v = 1.5$	A1	**3**	
(ii)	$0.6 = (c - b)/6$	M1		restitution (allow 1.5 for M1)
		A1		
	$6 \times 0.2 = 0.2b + 0.1c$	M1		momentum (allow 1.5 for M1)
		A1		
	$b = 2.8$	A1		
	$0.4 \times 5 + 0.2 \times 1.5 = 0.4a + 0.2 \times 6$ or $I = 0.9 = -0.4a - -0.4 \times 5$	M1		1st collision (needs their 1.5 for M1)
		A1		
	$a = 2.75$	A1		
	$2.75 < 2.8$	M1		compare v's of A and B (calculated)
	no further collision	A1	**10**	**13**

Q6, (Jan 2011, Q7)

(i)				
		$b + a = 1.8e$	M1	Uses restitution
			A1	$b - a = 1.8e$
			M1	Uses momentum
		$0.7b - 0.2a = 0.2 \times 1.8$	A1	$0.7b + 0.2a = 0.2 \times 1.8$, signs consistent with first eqn
			M1	Solves 2 simultaneous equations (eliminate a or b)
		$b = 0.4(1+e)$	A1	
		$a = 1.4e - 0.4$	A1	$a = 0.4 - 1.4e$
		$1.4e - 0.4 > 0.4 + 0.4e$	M1	Using a>b, correct signs in a essential
		$e > 0.8$	A1	
			[9]	
OR	**Last 5 marks**	Using $a > b$	M1	correct signs in a essential
		$a > 0.72$	A1	
		$b > 0.72$	A1	
		$1.8e > 0.72 + 0.72$	M1	
		$e > 0.8$	A1	
OR	**Last 5 marks**	Using $a = b$ to find a or b	M1	
		a (or b) = 0.9e and a (or b) = 0.72	A1	
		$e = 0.8$	A1	
		Convincing argument for correct inequality	M1	
		$e > 0.8$	A1	
OR	**Last 5 marks**		M1	Solves 2 simultaneous equations (eliminate a or b)
		$a = 1.4e - 0.4$ or $b = 0.4(1+e)$	A1	aef or multiples thereof
		Using $a > b$	M1	correct signs in a essential
		$a > 0.9e$ or $b < 0.9e$	A1	aef or multiples thereof
		$e > 0.8$	A1	

(ii)		c – (±0.25) = 1x0.75 c = 0.5, 1 0.75x0.7 = 0.25x0.7 + m (x1) *OR* 0.75x0.7 = -0.25x0.7 + 0.5m m = 0.35 (from first equation) m=1.4 (from second equation)	M1 A1A1 M1 A1 A1 **[6]**	Uses restitution with e = 1, either Or 0.75 ± 0.25 Uses momentum conservation with correct combination of sign and c value *OR* mx(0.75 ± 0.25) ± 0.7x0.25 = 0.75x0.7
OR		½ x0.7x0.75^2 = ½ x0.7X0.25^2 + ½mc^2 0.7x 0.75 = 0.7x(+/-0.25) + mc Solving simultaneous equations m = 0.35 m = 1.4	B1 M1 A1 M1 A1 A1	½ may not be seen At least one momentum equation mc = 0.35 and 0.7

Q7, (Jan 2013, Q3)

(i)		a = gsin30 $1+u = 0.4(2+2g\sin30)$ $u = 3.72$ ms^{-1}	B1 M1 A1 A1 **[4]**	 Using NEL with u_A from cv(a), $u_A \neq 0$ cwo
(ii)		Use $v^2 = u^2 - 2(g\sin30)s$ $s = 1.41$ m	M1 A1 **[2]**	Using v = 0, cv(a) from (i) or correct a SC If a not found in (i), allow a=g for M1A0.
(iii)		Use of conservation of momentum 0.5 x 2gsin30 - $2m = m$ - 0.5 x 3.72 $m = 2.25$	M1 A1ft A1 **[3]**	Using cv(a) ft cv(u) from (i) Aef(raction) eg 2 $^{19}/_{75}$ or $^{169}/_{75}$

Q8, (Jun 2014, Q6)

(i)		$(2m)(4) - (3m)(2) = 2mv_A + 3mv_B$	*M1 A1	Attempt at use of conservation of momentum
		$(v_B - v_A)/(4 - -2) = 0.4$	*M1 A1	Attempt at use of coefficient of restitution
		Speed $A = 1.04$ m s^{-1}, Speed $B = 1.36$ m s^{-1}	Dep**M1 A1 **[6]**	Solving for v_A and v_B Final answers must be positive
(ii)		Energy before = $\frac{1}{2}(2m)(4^2) + \frac{1}{2}(3m)(2^2)$ Energy after = $\frac{1}{2}(2m)(1.04^2) + \frac{1}{2}(3m)(1.36^2)$ $22m - 3.856m$ $18.1m$	B1ft B1ft M1 A1 **[4]**	Energy before or Loss in A's KE Energy after or Loss in B's KE Difference of total OR sum of differences (total kinetic energy must decrease) 18.144m (Exact)
	OR	$\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - e^2)A^2$ $\frac{1}{2}\frac{(2m)(3m)}{2m + 3m}(1 - 0.4^2)(4 + 2)^2$ $18.1m$	*B1 Dep*M1 A1 A1 **[4]**	Loss of kinetic energy formula, where A = approach speed Substitution of values into quoted formula 18.144m (Exact)
(iii)		$2m(4) - 2m(-1.04) = 2.52$ $m = 0.25$	M1 A1ft A1 **[3]**	Attempt at change in momentum and equate to impulse. Must use 2m or 3m Or $3m(2) - 3m(-1.36) = 2.52$ Exact

Q9, (Jun 2013, Q6)

(i)		M1	Use of restitution, may have sign errors, must be correct ratio (v/u)
	$4 - 4(1 - e + e^2) = -e(u - 4)$	A1	
	$u = 4e$	A1	oe
		M1	Use of conservation of momentum
	$mu + 0.2 \times 4 = 0.2 \times 4(1 - e + e^2) + 4m$	A1	
	$m = 0.2e$	A1	oe
		[6]	
(ii)	Valid method to find e that gives the least speed	M1	Differentiate v_A and equate to 0 or complete the square on v_A
	Get $e = ½$	A1	www
		M1	Difference of KE with 4 terms
	$½ \times 0.2 \times 4^2 + ½ \times 0.1 \times 2^2 - (½ \times 0.2 \times 3^2 + ½ \times 0.1 \times 4^2)$	A1	Must have found the value of e from a legitimate method. www
	(+/–) 0.1 J	A1	SCM1A1 Loss of KE = $8e(1 - e)^3/5$ or $8e(1 - 3e + 3e^2 - e^3)/5$ or $8e/5 - 24e^2/5 + 24e^3/5 - 8e^4/5$
		[5]	
(iii)		*M1	Attempt to use impulse = change in momentum on one particle
	$0.2e(4 - 4e) = 0.192$ or $0.2(4 - (4 - 4e + 4e^2)) = 0.192$	A1	
	Solve three term QE in e	dep*M1	method should lead to 2 real values for e
	$e = 0.4$ or 0.6	A1	For both
		[4]	

Q10, (Jun 2015, Q5)

(i)		$v^2 = 5^2 + 2g(1.6)$ $0.7 \times 7.507\ldots\ (= 5.255\ldots)$ $(0.7 \times \text{cv}(v))^2 = 2gh$ $h = 1.41$ m	B1 B1 M1 A1 **[4]**	Complete method to find v ($= 7.507\ldots$) $0.7 \times \text{cv}(v)$, **but not cv(v) = 5**; may be seen in (ii) Complete method to find h, with final speed 0; **allow cv(v) = 5** for method Exact 1.409
(ii)		$0.2(7.507\ldots)(0.7) - (-0.2)(7.507\ldots)$ Impulse = 2.55 N s, upwards	M1 A1ft A1 **[3]**	Change in momentum found, with relevant velocities ie cv(v) and $0.7 \times$ cv(v) but not cv(v) =5 This may be negative; ft on their v found in (i) (2.5524…) Must have direction also.
(iii)		$0.2(9.8)(1.6) + \frac{1}{2}(0.2)(5^2) - 0.2(9.8)(\text{cv}(h))$ OR $\frac{1}{2}(0.2)(7.507\ldots^2) - \frac{1}{2}(0.2)(0.7 \times 7.507\ldots)^2$ Loss of energy = 2.87 J	M1 A1ft A1 **[3]**	Change in energy found, all energy terms needed and no extra terms This may be negative (2.87436 exact); art 2.87; allow -2.87

Q11, (Jun 2016, Q6)

(i)		$4(8)+3(-10)=4v_A+3v_B$	M1*	Attempt at use of conservation of momentum
			A1	
		$\frac{1}{2}(4)(8)^2+\frac{1}{2}(3)(10)^2-\frac{1}{2}(4){v_A}^2-\frac{1}{2}(3){v_B}^2=121.5$	M1*	Attempt at use of KE(before) – KE (after) = 121.5
			A1	
			M1 dep*	Obtaining quadratic equation in either v_A or v_B $\left(7{v_B}^2-4v_B-416=0,\quad 28{v_A}^2-16v_A-935=0\right)$ and attempt to solve quadratic for either v_A or v_B
		$v_A=-5.5$ $(v_A=6.0714...)$ so speed of A is 5.5 (ms^{-1})	A1	cao; must be positive
		$v_B=8$ $(v_B=-7.428...)$ so speed of B is 8 (ms^{-1})	A1	cao; must be positive
		Both particles are moving in the reverse direction to their original motion	A1	Or an equivalent statement consistent with their v_A and v_B; left and right not sufficient without a diagram; moving away from each other needs a diagram also
			[8]	
(ii)		$v_A-v_B=-e(8-(-10))$	M1	Attempt at use of coefficient of restitution, right way round, v_A and v_B substituted
		$e=0.75$	A1	
			[2]	