FS1 (Year 2) – Quality of Hypothesis Tests Exam Questions (Edexcel)

Q1.

Sam and Tessa are testing a spinner to see if the probability, p, of it landing on red is less than $\frac{1}{5}$. They both use a 10% significance level.

Sam decides to spin the spinner 20 times and record the number of times it lands on red.

(a) Find the critical region for Sam's test.

(2)

(b) Write down the size of Sam's test.

(1)

Tessa decides to spin the spinner until it lands on red and she records the number of spins.

(c) Find the critical region for Tessa's test.

(6)

(d) Find the size of Tessa's test.

(1)

(e) (i) Show that the power function for Sam's test is given by

$$(1 - p)^{19}(1 + 19p)$$

(ii) Find the power function for Tessa's test.

(4)

(f) With reference to parts (b), (d) and (e), state, giving your reasons, whether you would recommend Sam's test or Tessa's test when $p = 0.15$

(4)

(Total for question = 18 marks)

(Q07 9FM0/3B-4B, Specimen papers)

Q2.

Some of the components produced by a factory are defective. The management requires that no more than 3% of the components produced are defective.

Niluki monitors the production process and takes a random sample of n components.

(a) Write down the hypotheses Niluki should use in a test to assess whether or not the proportion of defective components is greater than 0.03

(1)

Niluki defines the random variable D_n to represent the number of defective components in a sample of size n. She considers two tests **A** and **B**

In test **A**, Niluki uses $n = 100$ and if $D_{100} \geq 5$ she rejects H_0

(b) Find the size of test **A**

(2)

In test **B**, Niluki uses $n = 80$ and

- if $D_{80} \geq 5$ she rejects H_0
- if $D_{80} \leq 3$ she does not reject H_0
- if $D_{80} = 4$ she takes a second random sample of size 80 and if $D_{80} \geq 1$ in this second sample then she rejects H_0 otherwise she does not reject H_0

(c) Find the size of test **B**

(3)

Given that the actual proportion of defective components is 0.06

(d) (i) find the power of test **A**

(ii) find the expected number of components sampled using test **B**

(3)

Given also that, when the actual proportion of defective components is 0.06, the power of test **B** is 0.713

(e) suggest, giving your reasons, which test Niluki should use.

(1)

(Total for question = 10 marks)

(Q05 9FM0/03B, June 2024)

Q3.

Information was collected about accidents on the *Seapron* bypass. It was found that the number of accidents per month could be modelled by a Poisson distribution with mean 2.5

Following some work on the bypass, the numbers of accidents during a series of 3-month periods were recorded. The data were used to test whether or not there was a change in the mean number of accidents per month.

(a) Stating your hypotheses clearly and using a 5% level of significance, find the critical region for this test. You should state the probability in each tail.

(5)

(b) State P(Type I error) using this test.

(1)

Data from the series of 3-month periods are recorded for 2 years.

(c) Find the probability that at least 2 of these 3-month periods give a significant result.

(3)

Given that the number of accidents per month on the bypass, after the work is completed, is actually 2.1 per month,

(d) find P(Type II error) for the test in part (a)

(3)

(Total for question = 12 marks)

(Q05 9FM0/03B, June 2019)

Q4.

The number of tornadoes per year to hit a particular town follows a Poisson distribution with mean λ. A weatherman claims that due to climate changes the mean number of tornadoes per year has decreased. He records the number of tornadoes x to hit the town last year.

To test the hypotheses H_0: $\lambda = 7$ and H_1: $\lambda < 7$, a critical region of $x \leq 3$ is used.

(a) Find, in terms λ the power function of this test.

(3)

(b) Find the size of this test.

(2)

(c) Find the probability of a Type II error when $\lambda = 4$.

(2)

(Total 7 marks)

(Q02 6686/01, June 2007)

Q5.

The number of accidents per year in *Daftstown* follows a Poisson distribution with mean λ. The value of λ has previously been 6 but Jonty claims that since the Council increased the speed limit, the value of λ has increased.

Jonty records the number of accidents in *Daftstown* in the first year after the speed limit was increased. He plans to test, at the 5% significance level, whether or not there is evidence of an increase in the mean number of accidents in *Daftstown* per year.

(a) Stating your hypotheses clearly, calculate the probability of a Type I error for this test.

(4)

Given that there were 9 accidents in the first year after the speed limit was increased,

(b) state, giving a reason, whether or not there is evidence to support Jonty's claim.

(2)

(c) Given that the value of λ has actually increased to 8, calculate the probability of drawing the conclusion, using this test, that the number of accidents per year in *Daftstown* has not increased.

(2)

(Total for question = 8 marks)

(Q04 6686/01, June 2017)

Q6.

A jar contains a large number of sweets which have either soft centres or hard centres. The jar is thought to contain equal proportions of sweets with soft centres and sweets with hard centres. A random sample of 20 sweets is taken from the jar and the number of sweets with hard centres is recorded.

(a) Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that there are equal proportions of sweets with soft centres and sweets with hard centres in the jar.

(2)

(b) Calculate the probability of a Type I error for this test.

(2)

Given that there are 3 times as many sweets with soft centres as there are sweets with hard centres,

(c) calculate the probability of a Type II error for this test.

(2)

(Total for question = 6 marks)

(Q05 6686/01, June 2016)

Q7.

A manufacturer produces boxes of screws containing short screws and long screws. The manufacturer claims that the probability, p, of a randomly selected screw being long, is 0.5

A shopkeeper does not believe the manufacturer's claim. He designs two tests, A and B, to test the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$

In test A, a random sample of 10 screws is taken from a box of screws and H_0 is rejected if there are fewer than 3 long screws.

In test B, a random sample of 5 screws is taken from a box of screws and H_0 is rejected if there are no long screws, otherwise a second random sample of 5 screws is taken from a box of screws. If there are no long screws in this second sample H_0 is rejected, otherwise it is accepted.

(a) Find the size of test A.

(1)

(b) Find the size of test B.

(3)

(c) Find an expression for the power function of test B in terms of p.

(2)

Some values, to 2 decimal places, of the power function for test A and the power function for test B are given in the table below.

p	0.1	0.2	0.3	0.4
Power test A	0.93	r	0.38	0.17
Power test B	0.83	0.55	0.31	0.15

(d) Find the value of r.

(1)

The shopkeeper believes that the value of p is less than 0.4

(e) Suggest which of the tests the shopkeeper should use. Give a reason for your answer.

(2)

(Total for question = 9 marks)

(Q06 6686/01, June 2016)

Q8.

A poultry farm produces eggs which are sold in boxes of 6. The farmer believes that the proportion, p, of eggs that are cracked when they are packed in the boxes is approximately 5%. She decides to test the hypotheses

$$H_0 : p = 0.05 \quad \text{against} \quad H_1 : p > 0.05$$

To test these hypotheses she randomly selects a box of eggs and rejects H_0 if the box contains 2 or more eggs that are cracked. If the box contains 1 egg that is cracked, she randomly selects a second box of eggs and rejects H_0 if it contains at least 1 egg that is cracked. If the first or the second box contains no cracked eggs, H_0 is immediately accepted and no further boxes are sampled.

(a) Show that the power function of this test is

$$1 - (1 - p)^6 - 6p(1 - p)^{11}$$

(3)

(b) Calculate the size of this test.

(2)

Given that $p = 0.1$

(c) find the expected number of eggs inspected each time this test is carried out, giving your answer correct to 3 significant figures,

(3)

(d) calculate the probability of a Type II error.

(2)

Given that $p = 0.1$ is an unacceptably high value for the farmer,

(e) use your answer from part (d) to comment on the farmer's test.

(1)

(Total for question = 11 marks)

(Q02 6686/01, June 2015)

Q9.

(a) Define

(i) a Type I error,

(ii) a Type II error.

(2)

Rolls of material, manufactured by a machine, contain defects at a mean rate of 6 per roll.

The machine is modified. A single roll is selected at random and a test is carried out to see whether or not the mean number of defects per roll has decreased. The significance level is chosen to be as close as possible to 5%.

(b) Calculate the probability of a Type I error for this test.

(3)

(c) Given that the true mean number of defects per roll of material made by the machine is now 4, calculate the probability of a Type II error.

(2)

(Total 7 marks)

(Q04 6686/01, June 2014)

Q10.
A statistician believes a coin is biased and the probability, p, of getting a head when the coin is tossed is less than 0.5

The statistician decides to test this by tossing the coin 10 times and recording the number, X, of heads. He sets up the hypotheses $H_0 : p = 0.5$ and $H_1 : p < 0.5$ and rejects the null hypothesis if $x < 3$

(a) Find the size of the test.

(1)

(b) Show that the power function of this test is

$$(1 - p)^8 (36p^2 + 8p + 1)$$

(3)

Table 1 gives values, to 2 decimal places, of the power function for the statistician's test.

p	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Power	0.93	0.82	r	0.53	0.38	0.26	s	0.10

Table 1

(c) Calculate the value of r and the value of s.

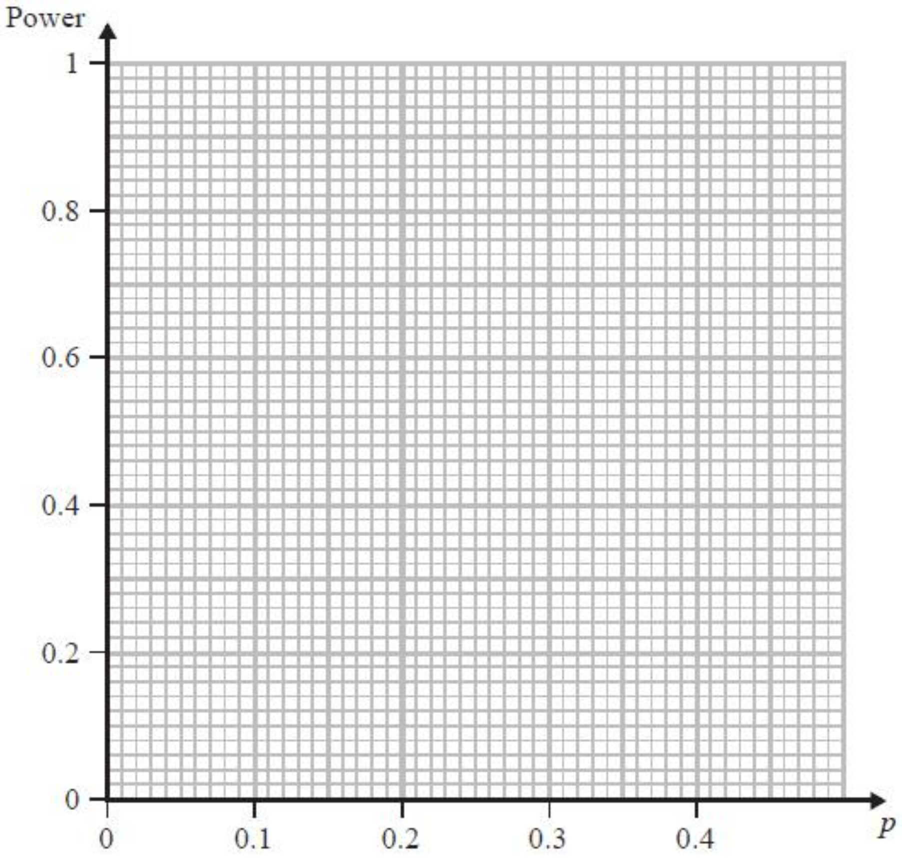
(2)

(d) On the axes below draw the graph of the power function for the statistician's test.

(2)

(e) Find the range of values of p for which the probability of accepting the coin as unbiased, when in fact it is biased, is less than or equal to 0.4

(3)



(Total 11 marks)

(Q05 6686/01, June 2014)

Q11.

Water is tested at various stages during a purification process by an environmental scientist. A certain organism occurs randomly in the water at a rate of λ every 10 ml. The scientist selects a random sample of 20 ml of water to check whether there is evidence that λ is greater than 1. The criterion the scientist uses for rejecting the hypothesis that $\lambda = 1$ is that there are 4 or more organisms in the sample of 20 ml.

(a) Find the size of the test.

(2)

(b) When $\lambda = 2.5$ find P(Type II error).

(2)

A statistician suggests using an alternative test. The statistician's test involves taking a random sample of 10 ml and rejecting the hypothesis that $\lambda = 1$ if 2 or more organisms are present but accepting the hypothesis if no organisms are in the sample. If only 1 organism is found then a second random sample of 10 ml is taken and the hypothesis is rejected if 2 or more organisms are present, otherwise the hypothesis is accepted.

(c) Show that the power of the statistician's test is given by

$$1 - e^{-\lambda} - \lambda(1 + \lambda)e^{-2\lambda}$$

(4)

Table 1 below gives some values, to 2 decimal places, of the power function of the statistician's test.

λ	1.5	2	2.5	3	3.5	4
Power	0.59	0.75	0.86	r	0.96	0.97

Table 1

(d) Find the value of r.

(1)

Figure 1 shows a graph of the power function for the scientist's test.

(e) On the same axes draw the graph of the power function for the statistician's test.

(2)

Given that it takes 20 minutes to collect and test a 20 ml sample and 15 minutes to collect and test a 10 ml sample

(f) show that the expected time of the statistician's test is slower than the scientist's test for $\lambda e^{-\lambda} > \frac{1}{3}$

(4)

(g) By considering the times when $\lambda = 1$ and $\lambda = 2$ together with the power curves in part (e) suggest, giving a reason, which test you would use.

(2)

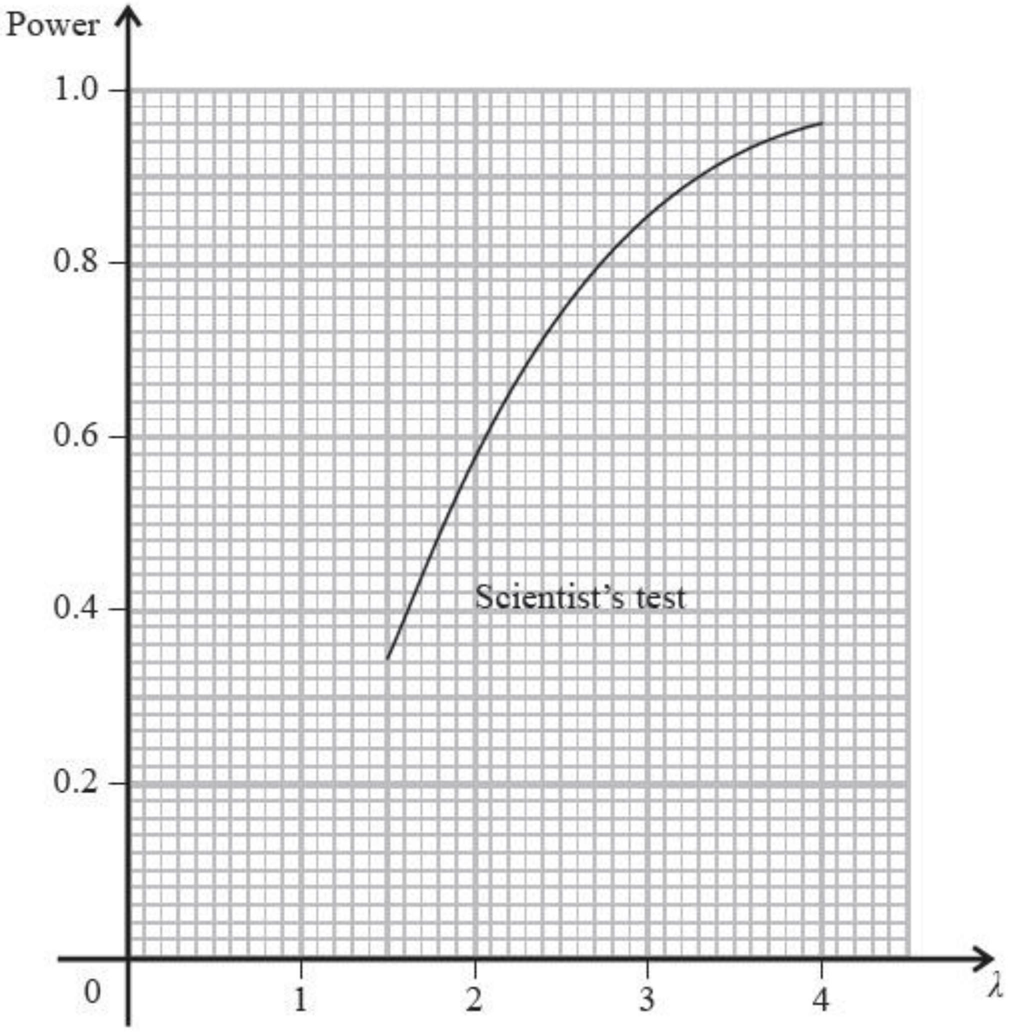


Figure 1

(Total 17 marks)

(Q02 6686/01, June 2013)

Q12.

The cloth produced by a certain manufacturer has defects that occur randomly at a constant rate of λ per square metre. If λ is thought to be greater than 1.5 then action has to be taken.

Using H_0:$\lambda = 1.5$ and H_1:$\lambda > 1.5$ a quality control officer takes a 4 m^2 sample of cloth and rejects H_0 if there are 11 or more defects. If there are 8 or fewer defects she accepts H_0. If there are 9 or 10 defects a second sample of 4 m^2 is taken and H_0 is rejected if there are 11 or more defects in this second sample, otherwise it is accepted.

(a) Find the size of this test.

(4)

(b) Find the power of this test when $\lambda = 2$

(3)

Q13.

A manager in a sweet factory believes that the machines are working incorrectly and the proportion p of underweight bags of sweets is more than 5%. He decides to test this by randomly selecting a sample of 5 bags and recording the number X that are underweight. The manager sets up the hypotheses H_0: $p = 0.05$ and H_1: $p > 0.05$ and rejects the null hypothesis if
$x > 1$.

(a) Find the size of the test.

(2)

(b) Show that the power function of the test is

$$1 - (1 - p)^4(1 + 4p)$$

(3)

The manager goes on holiday and his deputy checks the production by randomly selecting a sample of 10 bags of sweets. He rejects the hypothesis that $p = 0.05$ if more than 2 underweight bags are found in the sample.

(c) Find the probability of a Type I error using the deputy's test.

(2)

The table below gives some values, to 2 decimal places, of the power function for the deputy's test.

p	0.10	0.15	0.20	0.25
Power	0.07	s	0.32	0.47

(d) Find the value of s.

(1)

The graph of the power function for the manager's test is shown in Figure 1.

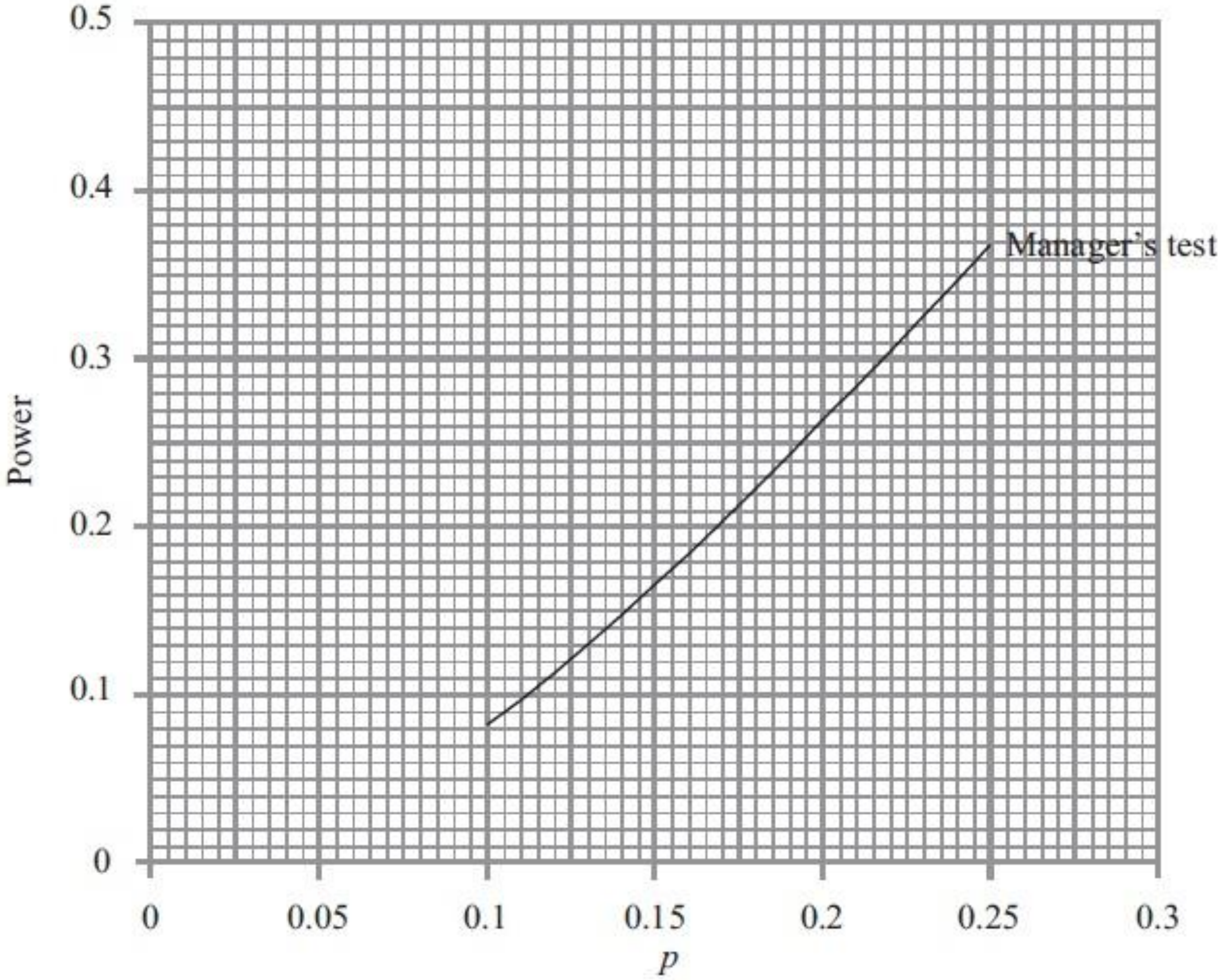


Figure 1

(e) On the same axes, draw the graph of the power function for the deputy's test.

(1)

(f) (i) State the value of p where these graphs intersect.

(ii) Compare the effectiveness of the two tests if p is greater than this value.

(2)

The deputy suggests that they should use his sampling method rather than the manager's.

(g) Give a reason why the manager might not agree to this change.

(1)

(Total 12 marks)

(**Q01 6686/01, June 2010**)

Q14.

Define, in terms of H_0 and/or H_1,

(a) the size of a hypothesis test,

(1)

(b) the power of a hypothesis test.

(1)

The probability of getting a head when a coin is tossed is denoted by p.

This coin is tossed 12 times in order to test the hypotheses H_0: $p = 0.5$ against H_1: $p \neq 0.5$, using a 5% level of significance.

(c) Find the largest critical region for this test, such that the probability in each tail is less than 2.5%.

(4)

(d) Given that $p = 0.4$

(i) find the probability of a type II error when using this test,

(ii) find the power of this test.

(4)

(e) Suggest two ways in which the power of the test can be increased.

(2)

(Total 12 marks)

(Q02 6686/01, June 2009)