FS1 (Year 2) Probability Generating Functions Exam Questions (Edexcel)

Q1.

The discrete random variable X has probability generating function

$$G_X(t) = k\ln\left(\frac{2}{2-t}\right)$$

where k is a constant.

(a) Find the exact value of k

(1)

(b) Find the exact value of Var(X)

(7)

(c) Find P($X = 3$)

(4)

(Total for question = 12 marks)

(Q06 9FM0/03B, June 2019)

Q2.

A discrete random variable X has probability generating function given by

$$G_X(t) = \frac{1}{64}(a + bt^2)^2$$

where a and b are positive constants.

(a) Write down the value of P($X = 3$)

(1)

Given that P($X = 4$) = $\frac{25}{64}$

(b) (i) find P($X = 2$)

(7)

(ii) find E(X)

(3)

The random variable $Y = 3X + 2$

(c) Find the probability generating function of Y

(2)

(Total for question = 13 marks)

(Q06 9FM0/03B, Oct 2020)

Q3.
The probability generating function of the random variable X is

$$G_X(t) = k(1 + 2t)^5$$

where k is a constant.

(a) Show that $k = \dfrac{1}{243}$

(2)

(b) Find $P(X = 2)$

(2)

(c) Find the probability generating function of $W = 2X + 3$

(2)

The probability generating function of the random variable Y is

$$G_Y(t) = \frac{t(1 + 2t)^2}{9}$$

Given that X and Y are independent,

(d) find the probability generating function of $U = X + Y$ in its simplest form.

(2)

(e) Use calculus to find the value of $\text{Var}(U)$

(6)

(Total for question = 14 marks)

(Q06 9FM0/03B, Oct 2021)

Q4.
The random variable X has probability generating function $G_X(t)$ where

$$G_X(t) = \frac{1}{\sqrt{4 - 3t}}$$

(a) Use calculus to find $\text{Var}(X)$

Show your working clearly.

(6)

(b) Find the exact value of $P(X \leq 2)$

(4)

The independent random variables X_1 and X_2 each have the same distribution as X

The random variable $Y = X_1 + X_2 + 1$

(c) By finding the probability generating function of Y, state the name of the distribution of Y

(4)

d) Hence, or otherwise, find $P(X_1 + X_2 > 5)$

(2)

(Total for question = 16 marks)

(Q06 9FM0/03B, June 2024)

Q5.

The probability generating function of the discrete random variable X is given by

$$G_X(t) = k(3 + t + 2t^2)^2$$

(a) Show that $k = \frac{1}{36}$

(2)

(b) Find $P(X = 3)$

(2)

(c) Show that $\text{Var}(X) = \frac{29}{18}$

(8)

(d) Find the probability generating function of $2X + 1$

(2)

(Total for question = 14 marks)

(Q06 9FM0/3B-4B, Specimen papers)

[Since so few exam questions on this topic exist, the following questions have been taken from the Cambridge Pre-U syllabus which seem to match the style and difficulty required]

Q6, (2013, Q5)

The discrete random variable X has probability generating function given by

$$G_X(t) = k(5t^{-1} + 3 + 2t^2),$$

where k is a constant.

(i) Find

(a) the value of k, [1]

[1]

(ii) The random variables X_1 and X_2 are independent observations of X.

(a) Write down the probability generating function of Y, where $Y = X_1 + X_2$. [1]

(b) Use your answer to part **(ii)(a)** to find $E(Y)$ and $\text{Var}(Y)$. [8]

Q7, (2015, Q3)

The probability generating function of the random variable X is $\frac{1}{81}\left(t + \frac{2}{t}\right)^4$.

(i) Use the probability generating function to find $E(X)$ and $\text{Var}(X)$. [5]

(ii) The random variable Y is defined by $Y = \frac{1}{2}(X + 4)$. By finding the probability distribution of X, or otherwise, show that $Y \sim B(n, p)$, stating the values of n and p. [4]

Q8, (2016, Q3)

(i) Show that the probability generating function of a random variable with the distribution $B(n, p)$ is $(1 - p + pt)^n$. [3]

(ii) R and S are independent random variables with the distributions $B(8, \frac{1}{4})$ and $B(8, \frac{3}{4})$ respectively. Show that the probability generating function of $R + S$ can be expressed as

$$\left(\frac{3}{16} + \frac{1}{16}t(10 + 3t)\right)^8$$

and use this result to find $P(R + S = 1)$. [5]

Q9, (2017, Q2)

A discrete random variable X has the following probability distribution.

x	-1	2
$P(X = x)$	$\frac{1}{3}$	$\frac{2}{3}$

(i) Write down the probability generating function of X. [2]

(ii) T is the sum of ten independent observations of X. Use the probability generating function of T to find

(a) $E(T)$, [4]

(b) $P(T = 8)$. [3]

Q10, (2019 Specimen, Q1)

The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = at\left(t + \frac{1}{t}\right)^3,$$

where a is a constant.

(a) Find, in either order, the value of a and the set of values that X can take. [4]

(b) Find the value of $E(X)$. [2]
