Poisson Approximations to a Binomial Distribution (From Edexcel 6684)

Q1, (Jun 2006, Q5)

A manufacturer produces large quantities of coloured mugs. It is known from previous records that 6% of the production will be green.

A random sample of 10 mugs was taken from the production line.

(a) Define a suitable distribution to model the number of green mugs in this sample.

(1)

(b) Find the probability that there were exactly 3 green mugs in the sample.

(3)

A random sample of 125 mugs was taken.

(c) Find the probability that there were between 10 and 13 (inclusive) green mugs in this sample, using

(i) a Poisson approximation,

(3)

(ii) a Normal approximation.

(6)

Q2, (Jan 2008, Q6)

The probability that a sunflower plant grows over 1.5 metres high is 0.25. A random sample of 40 sunflower plants is taken and each sunflower plant is measured and its height recorded.

(a) Find the probability that the number of sunflower plants over 1.5 m high is between 8 and 13 (inclusive) using

(i) a Poisson approximation,

(ii) a Normal approximation.

(10)

(b) Write down which of the approximations used in part (a) is the most accurate estimate of the probability. You must give a reason for your answer.

(2)

Q3, (Jun 2007, Q5)

(a) Write down the conditions under which the Poisson distribution may be used as an approximation to the Binomial distribution.

(2)

A call centre routes incoming telephone calls to agents who have specialist knowledge to deal with the call. The probability of the caller being connected to the wrong agent is 0.01

(b) Find the probability that 2 consecutive calls will be connected to the wrong agent.

(2)

(c) Find the probability that more than 1 call in 5 consecutive calls are connected to the wrong agent.

(3)

The call centre receives 1000 calls each day.

(d) Find the mean and variance of the number of wrongly connected calls.

(3)

(e) Use a Poisson approximation to find, to 3 decimal places, the probability that more than 6 calls each day are connected to the wrong agent.

(2)

Q4, (Jan 2009, Q5)

A factory produces components of which 1% are defective. The components are packed in boxes of 10. A box is selected at random.

(a) Find the probability that the box contains exactly one defective component.

(2)

(b) Find the probability that there are at least 2 defective components in the box.

(3)

(c) Using a suitable approximation, find the probability that a batch of 250 components contains between 1 and 4 (inclusive) defective components.

(4)

Q5, (Jun 2010, Q2)

Bhim and Joe play each other at badminton and for each game, independently of all others, the probability that Bhim loses is 0.2

Find the probability that, in 9 games, Bhim loses

(a) exactly 3 of the games,

(3)

(b) fewer than half of the games.

(2)

Bhim attends coaching sessions for 2 months. After completing the coaching, the probability that he loses each game, independently of all others, is 0.05

Bhim and Joe agree to play a further 60 games.

(c) Calculate the mean and variance for the number of these 60 games that Bhim loses.

(2)

(d) Using a suitable approximation calculate the probability that Bhim loses more than 4 games.

(3)

Q6, (Jan 2012, Q5)

The probability of an electrical component being defective is 0.075
The component is supplied in boxes of 120

(a) Using a suitable approximation, estimate the probability that there are more than 3 defective components in a box.

(5)

A retailer buys 2 boxes of components.

(b) Estimate the probability that there are at least 4 defective components in each box.

(2)

Q6, (Jan 2011, Q1)

A disease occurs in 3% of a population.

(a) State any assumptions that are required to model the number of people with the disease in a random sample of size n as a binomial distribution.

(2)

(b) Using this model, find the probability of exactly 2 people having the disease in a random sample of 10 people.

(3)

(c) Find the mean and variance of the number of people with the disease in a random sample of 100 people.

(2)

A doctor tests a random sample of 100 patients for the disease. He decides to offer all patients a vaccination to protect them from the disease if more than 5 of the sample have the disease.

(d) Using a suitable approximation, find the probability that the doctor will offer all patients a vaccination.

(3)

Q8, (Jun 2012, Q3)

(a) Write down two conditions needed to approximate the binomial distribution by the Poisson distribution.

(2)

A machine which manufactures bolts is known to produce 3% defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts were defective.

(b) Using a suitable approximation, test at the 5% level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.

(7)
