Hypothesis Tests for Mean of a Poisson Distribution (From Edexcel 6684)

Q1, (Jun 2006, Q4)

Breakdowns occur on a particular machine at random at a mean rate of 1.25 per week.

(a) Find the probability that fewer than 3 breakdowns occurred in a randomly chosen week.

(4)

Over a 4 week period the machine was monitored. During this time there were 11 breakdowns.

(b) Test, at the 5% level of significance, whether or not there is evidence that the rate of breakdowns has changed over this period. State your hypotheses clearly.

(7)

Q2, (Jun 2007, Q2)

Bacteria are randomly distributed in a river at a rate of 5 per litre of water. A new factory opens and a scientist claims it is polluting the river with bacteria. He takes a sample of 0.5 litres of water from the river near the factory and finds that it contains 7 bacteria. Stating your hypotheses clearly test, at the 5% level of significance, the claim of the scientist.

(7)

Q3, (Jn 2008, Q3)

A test statistic has a Poisson distribution with parameter λ.

Given that

$$H_0 : \lambda = 9,\ H_1 : \lambda \neq 9$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%.

(3)

(b) State the probability of incorrectly rejecting H_0 using this critical region.

(2)

Q4, (Jun 2009, Q2)

An effect of a certain disease is that a small number of the red blood cells are deformed. Emily has this disease and the deformed blood cells occur randomly at a rate of 2.5 per ml of her blood. Following a course of treatment, a random sample of 2 ml of Emily's blood is found to contain only 1 deformed red blood cell.

Stating your hypotheses clearly and using a 5% level of significance, test whether or not there has been a decrease in the number of deformed red blood cells in Emily's blood.

(6)

Q5, (Jan 2011, Q4)

Richard regularly travels to work on a ferry. Over a long period of time, Richard has found that the ferry is late on average 2 times every week. The company buys a new ferry to improve the service. In the 4-week period after the new ferry is launched, Richard finds the ferry is late 3 times and claims the service has improved. Assuming that the number of times the ferry is late has a Poisson distribution, test Richard's claim at the 5% level of significance. State your hypotheses clearly.

(6)

Q6, (Jun 20154, Q3)

A company claims that it receives emails at a mean rate of 2 every 5 minutes.

(a) Give two reasons why a Poisson distribution could be a suitable model for the number of emails received.

(2)

(b) Using a 5% level of significance, find the critical region for a two-tailed test of the hypothesis that the mean number of emails received in a 10 minute period is 4. The probability of rejection in each tail should be as close as possible to 0.025

(2)

(c) Find the actual level of significance of this test.

(2)

To test this claim, the number of emails received in a random 10 minute period was recorded.

During this period 8 emails were received.

(d) Comment on the company's claim in the light of this value. Justify your answer.

(2)

During a randomly selected 15 minutes of play in the Wimbledon Men's Tennis Tournament final, 2 emails were received by the company.

(e) Test, at the 10% level of significance, whether or not the mean rate of emails received by the company during the Wimbledon Men's Tennis Tournament final is lower than the mean rate received at other times. State your hypotheses clearly.

(5)

Q7, (Jun 2015, Q5)

Liftsforall claims that the lift they maintain in a block of flats breaks down at random at a mean rate of 4 times per month. To test this, the number of times the lift breaks down in a month is recorded.

(a) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis that 'the mean rate at which the lift breaks down is 4 times per month'. The probability of rejection in each of the tails should be as close to 2.5% as possible.

(3)

Over a randomly selected 1 month period the lift broke down 3 times.

(b) Test, at the 5% level of significance, whether *Liftsforall*'s claim is correct. State your hypotheses clearly.

(2)

(c) State the actual significance level of this test.

(1)

The residents in the block of flats have a maintenance contract with *Liftsforall*. The residents pay *Liftsforall* £500 for every quarter (3 months) in which there are at most 3 breakdowns. If there are 4 or more breakdowns in a quarter then the residents do not pay for that quarter.

Liftsforall installs a new lift in the block of flats.

Given that the new lift breaks down at a mean rate of 2 times per month,

(d) find the probability that the residents do not pay more than £500 to *Liftsforall* in the next year.

(6)