Geometric Distribution (From OCR 4732)

Q1, (Jan 2006, Q5)

Andrea practises shots at goal. For each shot the probability of her scoring a goal is $\frac{2}{5}$. Each shot is independent of other shots.

(i) Find the probability that she scores her first goal

(a) on her 5th shot, [2]

(b) before her 5th shot. [3]

(ii) (a) Find the probability that she scores exactly 1 goal in her first 5 shots. [3]

(b) Hence find the probability that she scores her **second** goal on her 6th shot. [2]

Q2, (Jan 2007, Q6)

A coin is biased so that the probability that it will show heads on any throw is $\frac{2}{3}$. The coin is thrown repeatedly.

The number of throws up to and including the first head is denoted by X. Find

(i) $P(X = 4)$, [3]

(ii) $P(X < 4)$, [3]

(iii) $E(X)$. [2]

Q3, (Jan 2008, Q2)

A random variable T has the distribution $\text{Geo}(\frac{1}{5})$. Find

(i) $P(T = 4)$, [2]

(ii) $P(T > 4)$, [2]

(iii) $E(T)$. [1]

Q4, (Jan 2009, Q3)

Erika is a birdwatcher. The probability that she will see a woodpecker on any given day is $\frac{1}{8}$. It is assumed that this probability is unaffected by whether she has seen a woodpecker on any other day.

(i) Calculate the probability that Erika first sees a woodpecker

(a) on the third day, **[3]**

(b) after the third day. **[3]**

(ii) Find the expectation of the number of days up to and including the first day on which she sees a woodpecker. **[1]**

(iii) Calculate the probability that she sees a woodpecker on exactly 2 days in the first 15 days. **[3]**

Q5, (Jun 2009, Q4)

30% of people own a Talk-2 phone. People are selected at random, one at a time, and asked whether they own a Talk-2 phone. The number of people questioned, up to and including the first person who owns a Talk-2 phone, is denoted by X. Find

(i) $P(X = 4)$, **[3]**

(ii) $P(X > 4)$, **[2]**

(iii) $P(X < 6)$. **[3]**

Q6, (Jan 2011, Q2)

The random variable X has the distribution Geo(0.2). Find

(i) $P(X = 3)$, **[2]**

(ii) $P(3 \leqslant X \leqslant 5)$, **[3]**

(iii) $P(X > 4)$. **[3]**

Two independent values of X are found.

(iv) Find the probability that the total of these two values is 3. **[3]**

Q7, (Jun 2011, Q8)

Ann, Bill, Chris and Dipak play a game with a fair cubical die. Starting with Ann they take turns, in alphabetical order, to throw the die. This process is repeated as many times as necessary until a player throws a 6. When this happens, the game stops and this player is the winner.

Find the probability that

(i) Chris wins on his first throw, **[1]**

(ii) Dipak wins on his second throw, **[3]**

(iii) Ann gets a third throw, **[2]**

(iv) Bill throws the die exactly three times. **[4]**

Q8, (Jun 2012, Q9)

(i) A clock is designed to chime once each hour, on the hour. The clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{10}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day. Find the probability that the first time it does not chime is

(a) at 0600 on that day, **[3]**

(b) before 0600 on that day. **[3]**

(ii) Another clock is designed to chime twice each hour: on the hour and at 30 minutes past the hour. This clock has a fault so that each time it is supposed to chime there is a constant probability of $\frac{1}{20}$ that it will not chime. It may be assumed that the clock never stops and that faults occur independently. The clock is started at 5 minutes past midnight on a certain day.

(a) Find the probability that the first time it does not chime is at either 0030 or 0130 on that day. **[2]**

(b) Use the formula for the sum to infinity of a geometric progression to find the probability that the first time it does not chime is at 30 minutes past some hour. **[3]**

Q9, (Jan 2013, Q8)

Sandra makes repeated, independent attempts to hit a target. On each attempt, the probability that she succeeds is 0.1.

(i) Find the probability that

(a) the first time she succeeds is on her 5th attempt, **[2]**

(b) the first time she succeeds is after her 5th attempt, **[2]**

(c) the second time she succeeds is before her 4th attempt. **[4]**

Jill also makes repeated attempts to hit the target. Each attempt of either Jill or Sandra is independent. Each time that Jill attempts to hit the target, the probability that she succeeds is 0.2. Sandra and Jill take turns attempting to hit the target, with Sandra going first.

(ii) Find the probability that the first person to hit the target is Sandra, on her

(a) 2nd attempt, **[2]**

(b) 10th attempt. **[3]**

Q10, (Jun 2013, Q9)

A game is played with a token on a board with a grid printed on it. The token starts at the point (0, 0) and moves in steps. Each step is either 1 unit in the positive x-direction with probability 0.8, or 1 unit in the positive y-direction with probability 0.2. The token stops when it reaches a point with a y-coordinate of 1. It is given that the token stops at $(X, 1)$.

(i) (a) Find the probability that $X = 10$. **[2]**

(b) Find the probability that $X < 10$. **[3]**

(ii) Find the expected number of steps taken by the token. **[2]**

(iii) Hence, write down the value of $\mathrm{E}(X)$. **[1]**

Q11, (Jun 2014, Q9)

Each day Harry makes repeated attempts to light his gas fire. If the fire lights he makes no more attempts. On each attempt, the probability that the fire will light is 0.3 independent of all other attempts. Find the probability that

(i) the fire lights on the 5th attempt, **[2]**

(ii) Harry needs more than 1 attempt but fewer than 5 attempts to light the fire. **[3]**

If the fire does not light on the 6th attempt, Harry stops and the fire remains unlit.

(iii) Find the probability that, on a particular day, the fire lights. **[3]**

(iv) Harry's week starts on Monday. Find the probability that, during a certain week, the first day on which the fire lights is Wednesday. **[2]**

Q12, (Jun 2015, Q5)

Each year Jack enters a ballot for a concert ticket. The probability that Jack will win a ticket in any particular year is 0.27.

(i) Find the probability that the first time Jack wins a ticket is

(a) on his 8th attempt, **[2]**

(b) after his 8th attempt. **[2]**

(ii) Write down an expression for the probability that Jack wins a ticket on exactly 2 of his first 8 attempts, and evaluate this expression. **[3]**

(iii) Find the probability that Jack wins his 3rd ticket on his 9th attempt and his 4th ticket on his 12th attempt. **[3]**

Q13, (Jun 2016, Q7)

On average Marie scores a goal on 20% of her shots. The variable random X is the number of shots Marie takes, up to and including her first goal.

(i) State two conditions needed for X to have a geometric distribution. **[2]**

(ii) Assuming these conditions are satisfied, find the probability that

(a) $X = 3$, **[2]**

(b) $X < 10$, **[2]**

(c) $9 < X < 20$. **[3]**

The probability that Nadine scores a goal on any shot is 0.3. Marie and Nadine independently take shots in turn, with Marie shooting first. The winner is the first one to score two goals.

(iii) Find the probability that

(a) Marie wins on her second shot, **[2]**

(b) Nadine wins on her second shot. **[3]**
