Expectation and Variance of Linear Transformations of Discrete Random Variables (From Edexcel 6683)

Q1, (Jun 2006, Q4)

The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \qquad x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $E(X)$ and show that $Var(X) = 2$.

(3)

Find

(b) $E(3X - 2)$,

(2)

(c) $Var(4 - 3X)$.

(2)

Q2, (Jun 2007, Q7)

The random variable X has probability distribution

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

(a) Given that $E(X) = 4.5$, write down two equations involving p and q.

(3)

Find

(b) the value of p and the value of q,

(3)

(c) $P(4 < X \leqslant 7)$.

(2)

Given that $E(X^2) = 27.4$, find

(d) $Var(X)$,

(2)

(e) $E(19 - 4X)$,

(1)

(f) $Var(19 - 4X)$.

(2)

Q3, (Jun 2009, Q6)

The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3-x) & x = 0,1,2 \\ b & x = 3 \end{cases}$$

(a) Find $P(X = 2)$ and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that $E(X) = 1.6$

(b) Find the value of a and the value of b.

(5)

Find

(c) $P(0.5 < X < 3)$,

(2)

(d) $E(3X - 2)$.

(2)

(e) Show that the $Var(X) = 1.64$

(3)

(f) Calculate $Var(3X - 2)$.

(2)

Q4, (Jun 2013, Q5a-c)

A biased die with six faces is rolled. The discrete random variable X represents the score on the uppermost face. The probability distribution of X is shown in the table below.

x	1	2	3	4	5	6
$P(X = x)$	a	a	a	b	b	0.3

(a) Given that $E(X) = 4.2$ find the value of a and the value of b.

(5)

(b) Show that $E(X^2) = 20.4$

(1)

(c) Find $Var(5 - 3X)$

(3)

Q5, (Jun 2012, Q1)

A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1, 0, 1 \text{ and } 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{1}{6}$

(3)

(b) Find $E(X)$

(2)

(c) Show that $E(X^2) = \frac{4}{3}$

(2)

(d) Find $\text{Var}(1 - 3X)$

(3)

Q6, (Jun 2013(R), Q7a-e)

The score S when a spinner is spun has the following probability distribution.

s	0	1	2	4	5
$P(S = s)$	0.2	0.2	0.1	0.3	0.2

(a) Find $E(S)$.

(2)

(b) Show that $E(S^2) = 10.4$

(2)

(c) Hence find $\text{Var}(S)$.

(2)

(d) Find

(i) $E(5S - 3)$,

(ii) $\text{Var}(5S - 3)$.

(4)

(e) Find $P(5S - 3 > S + 3)$

(3)

Q7, (Jan 2010, Q5)

The probability function of a discrete random variable X is given by

$$p(x) = kx^2 \qquad x = 1, 2, 3$$

where k is a positive constant.

(a) Show that $k = \frac{1}{14}$

(2)

Find

(b) $P(X \geqslant 2)$

(2)

(c) $E(X)$

(2)

(d) $\text{Var}(1 - X)$

(4)