D1 (Year 2) – Travelling Salesman Problem Exam Questions (Edexcel)

Q1.

(a) Explain clearly the difference between the classical travelling salesperson problem and the practical travelling salesperson problem.

(2)

	A	B	C	D	E	F	G
A	-	17	24	16	21	18	41
B	17	-	35	25	30	31	x
C	24	35	-	28	20	35	32
D	16	25	28	-	29	19	45
E	21	30	20	29	-	22	35
F	18	31	35	19	22	-	37
G	41	x	32	45	35	37	-

The table shows the least distances, in km, by road between seven towns, A, B, C, D, E, F and G. The least distance between B and G is x km, where $x > 25$

Preety needs to visit each town at least once, starting and finishing at A. She wishes to minimise the total distance she travels.

(b) Starting by deleting B and all of its arcs, find a lower bound for Preety's route.

(3)

Preety found the nearest neighbour routes from each of A and C. Given that the sum of the lengths of these routes is 331 km,

(c) find x, making your method clear.

(4)

(d) Write down the smallest interval that you can be confident contains the optimal length of Preety's route. Give your answer as an inequality.

(2)

(Total for question = 11 marks)

(Q03 9FM0/3D-4D, Specimen papers)

Q2.

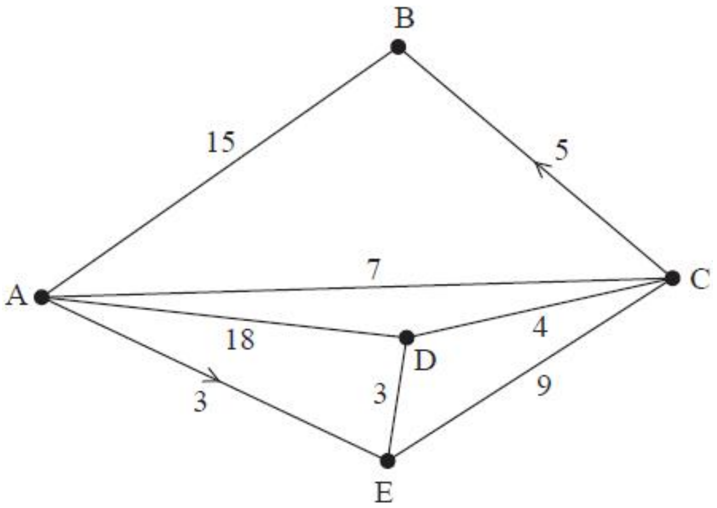


Figure 2

The network in Figure 2 shows the direct roads linking five villages, A, B, C, D and E.
The number on each arc represents the length, in miles, of the corresponding road.
The roads from A to E and from C to B are one-way, as indicated by the arrows.

(a) Complete the initial distance and route tables for the network provided in the answer book.

(2)

(b) Perform the first three iterations of Floyd's algorithm. You should show the distance table and the route table after each of the three iterations.

(5)

After five iterations of Floyd's algorithm the final distance table and partially completed final route table are shown below.

Distance table

	A	**B**	**C**	**D**	**E**
A	–	12	7	6	3
B	15	–	22	21	18
C	7	5	–	4	7
D	11	9	4	–	3
E	14	12	7	3	–

Route table

	A	**B**	**C**	**D**	**E**
A	A				
B	A	B			
C	A	B	C		
D	C	C	C	D	
E	D	D	D	D	E

(c) (i) Explain how the partially completed final route table can be used to find the shortest route from E to A.

(ii) State this route.

(3)

Mabintou decides to use the distance table to try to find the shortest cycle that passes through each vertex. Starting at D, she applies the nearest neighbour algorithm to the final distance table.

(d) (i) State the cycle obtained using the nearest neighbour algorithm.

(ii) State the length of this cycle.
(iii) Interpret the cycle in terms of the actual villages visited.
(iv) Prove that Mabintou's cycle is not optimal.

(4)

(Total for question = 14 marks)

(Q03 9FM0/03D, June 2019)

Q3.

	A	B	C	D	E	F
A	–	83	75	82	69	97
B	83	–	94	103	77	109
C	75	94	–	97	120	115
D	82	103	97	–	105	125
E	69	77	120	105	–	88
F	97	109	115	125	88	–

The table above shows the least distances, in km, between six towns, A, B, C, D, E and F.

(a) Starting at A, and making your working clear, find an initial upper bound for the travelling salesperson problem for this network, using

(i) the minimum spanning tree method,
(ii) the nearest neighbour algorithm.

(5)

By deleting A, and all of its arcs, a lower bound for the travelling salesperson problem for this network is found to be 500 km.

By deleting B, and all of its arcs, the corresponding lower bound is found to be 474 km.

(b) Using the results from (a) and the given lower bounds, write down the smallest interval that you can be confident contains the solution to the travelling salesperson problem for this network.

(2)

(Total for question = 7 marks)

(Q07 6690/01, June 2017)

Q4.

	A	B	C	D	E	F	G
A	–	31	15	12	24	17	22
B	31	–	20	25	14	25	50
C	15	20	–	16	24	19	21
D	12	25	16	–	21	32	17
E	24	14	24	21	–	28	41
F	17	25	19	32	28	–	25
G	22	50	21	17	41	25	–

(a) Explain the difference between the classical travelling salesperson problem and the practical travelling salesperson problem.

(2)

The table above shows the least direct distances, in miles, between seven towns, A, B, C, D, E, F and G. Yiyi needs to visit each town, starting and finishing at A, and wishes to minimise the total distance she will travel.

(b) Show that there are two nearest neighbour routes that start from A. State these routes and their lengths.

(3)

(c) Starting by deleting A, and all of its arcs, find a lower bound for the length of Yiyi's route.

(3)

(d) Use your results to write down the smallest interval which you can be confident contains the optimal length of Yiyi's route.

(2)

(Total for question = 10 marks)

(Q08 6690/01, June 2016)

Q5.

	A	B	C	D	E	F	G
A	–	x	41	43	38	21	30
B	x	–	27	38	19	29	51
C	41	27	–	24	37	35	40
D	43	38	24	–	44	52	25
E	38	19	37	44	–	20	28
F	21	29	35	52	20	–	49
G	30	51	40	25	28	49	–

The network represented by the table shows the least distances, in km, between seven theatres, A, B, C, D, E, F and G.

Jasmine needs to visit each theatre at least once starting and finishing at A. She wishes to minimise the total distance she travels. The least distance between A and B, is x km, where $21 < x < 27$

(a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You should list the arcs in the order in which you consider them.

(2)

(b) Use your answer to (a) to determine an initial upper bound for the length of Jasmine's route.

(1)

(c) Use the nearest neighbour algorithm, starting at A, to find a second upper bound for the length of the route.

(2)

The nearest neighbour algorithm starting at F gives a route of F – E – B – A – G – D – C – F.

(d) State which of these two nearest neighbour routes gives the better upper bound. Give a reason for your answer.

(2)

Starting by deleting A, and all of its arcs, a lower bound of 159 km for the length of the route is found.

(e) Find x, making your method clear.

(3)

(f) Write down the smallest interval that you can be confident contains the optimal length of Jasmine's route. Give your answer as an inequality.

(2)

(Total for question = 12 marks)
(Q07 6690/01, June 2015)

Q6.

	A	B	C	D	E	F
A	–	65	48	15	30	40
B	65	–	50	51	35	26
C	48	50	–	37	20	34
D	15	51	37	–	17	25
E	30	35	20	17	–	14
F	40	26	34	25	14	–

(a) Explain the difference between the classical and the practical travelling salesperson problem.

(2)

The table above shows the least distances, in km, between six towns, A, B, C, D, E and F. Keith needs to visit each town, starting and finishing at A, and wishes to minimise the total distance he will travel.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound. You must state your route and its length.

(3)

(c) Starting by deleting A, and all of its arcs, find a lower bound for the route length.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(2)

(Total 10 marks)

(Q08 6690/01/R, June 2014)

Q7.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

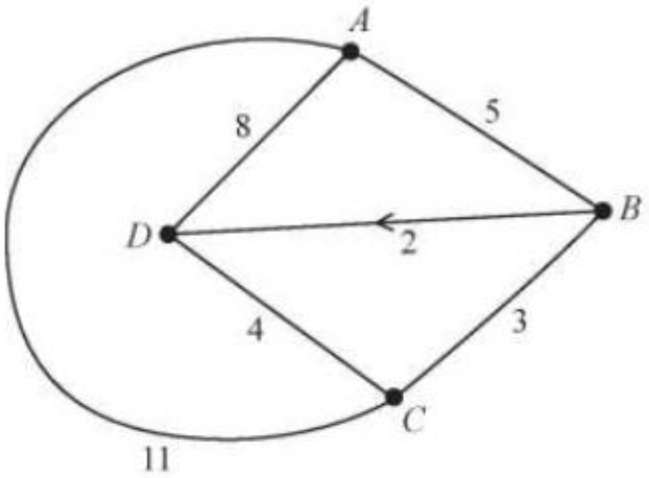
(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total 8 marks)

(Q07 6690/01/R, June 2013)

Q8.



The network in Figure 3 shows the roads linking a depot, D, and three collection points A, B and C. The number on each arc represents the length, in miles, of the corresponding road. The road from B to D is a one-way road, as indicated by the arrow.

(a) Explain clearly if Dijkstra's algorithm can be used to find a route from D to A.

(1)

The initial distance and route tables for the network are given in the answer book.

(b) Use Floyd's algorithm to find a table of least distances. You should show both the distance table and the route table after each iteration.

(7)

(c) Explain how the final route table can be used to find the shortest route from D to B. State this route.

(2)

There are items to collect at A, B and C. A van will leave D to make these collections in any order and then return to D. A minimum route is required.
Using the final distance table and the Nearest Neighbour algorithm starting at D,

(d) find a minimum route and state its length.

(2)

Floyd's algorithm and Dijkstra's algorithm are applied to a network. Each will find the shortest distance between vertices of the network.

(e) Describe how the results of these algorithms differ.

(2)

(Total for question = 14 marks)

(Q04 9FM0/3D-4D, Specimen papers)

Q9.

(a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between six data collection points, A, B, C, D, E, and F.

	A	B	C	D	E	F
A	-	77	34	56	67	21
B	77	-	58	58	36	74
C	34	58	-	73	70	42
D	56	58	73	-	68	38
E	67	36	70	68	-	71
F	21	74	42	38	71	-

Rachel must visit each collection point. She will start and finish at A and wishes to minimise the total distance travelled.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound. Make your method clear.

(3)

Starting at B, a second upper bound of 293 km was found.

(c) State the better upper bound of these two, giving a reason for your answer.

(1)

By deleting A, a lower bound was found to be 245 km.

(d) By deleting B, find a second lower bound. Make your method clear.

(4)

(e) State the better lower bound of these two, giving a reason for your answer.

(1)

(f) Taking your answers to (c) and (e), use inequalities to write down an interval that must contain the length of Rachel's optimal route.

(1)

(Total 12 marks)

(Q07 6690/01, June 2009)

Q10.

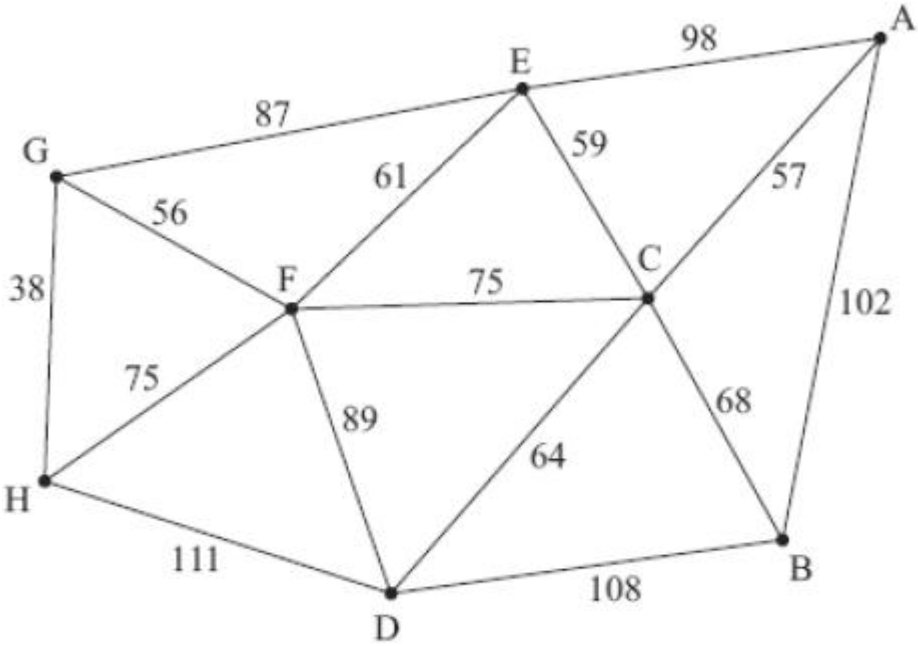


Figure 2

The network in figure 2 shows the distances, in km, between eight weather data collection points. Starting and finishing at A, Alice needs to visit each collection point at least once, in a minimum distance.

(a) Obtain a minimum spanning tree for the network using Kruskal's algorithm, stating the order in which you select the arcs.

(2)

(b) Use your answer to part (a) to determine an initial upper bound for the length of the route.

(1)

(c) Starting from your initial upper bound use short cuts to find an upper bound, which is below 630km. State the corresponding route.

(4)

(d) Use the nearest neighbour algorithm starting at B to find a second upper bound for the length of the route.

(3)

(e) By deleting C, and all of its arcs, find a lower bound for the length of the route.

(4)

(f) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(2)

(Total 16 marks)

(Q08 6690/01, June 2008)