D1 (Year 2) – Simplex Method (Two-Stage and Big-M) Exam Questions (Edexcel)

Q1.
Susie is preparing for a triathlon event that is taking place next month. A triathlon involves three activities: swimming, cycling and running.

Susie decides that in her training next week she should

- maximise the total time spent cycling and running
- train for at most 39 hours
- spend at least 40% of her time swimming
- spend a total of at least 28 hours of her time swimming and running

Susie needs to determine how long she should spend next week training for each activity. Let

- x represent the number of hours swimming
- y represent the number of hours cycling
- z represent the number of hours running

(a) Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(5)

Susie decides to solve this linear programming problem by using the two-stage Simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage Simplex method.

As part of your solution you must show how

- the constraints have been made into equations using slack variables, exactly one surplus variable and exactly one artificial variable
- the rows for the two objective functions are formed

(6)

The following tableau T is obtained after one iteration of the second stage of the two-stage Simplex method.

b.v.	x	y	z	s_1	s_2	s_3	Value
y	0	1	0	1	0	1	11
s_2	0	0	5	−2	1	−5	62
x	1	0	1	0	0	−1	28
P	0	0	−1	1	0	1	11

(c) Obtain a suitable pivot for a second iteration. You must give reasons for your answer.

(2)

(d) Starting from tableau T, solve the linear programming problem by performing one further iteration of the second stage of the two-stage Simplex method. You should make your method clear by stating the row operations you use.

(5)

(Total for question = 18 marks)

(Q08 9FM0/03D, Oct 2021)

Q2.
A publisher plans to produce three versions of the same book: a paperback, a hardcover, and a deluxe edition.

- Each paperback takes 4 minutes to print and 1 minute to bind
- Each hardcover takes 8 minutes to print and 5 minutes to bind
- Each deluxe edition takes 15 minutes to print and 12 minutes to bind

The printing machine is available for at most 150 hours and the binding machine must be used for at least 60 hours.

The publisher decides to produce

- at least 1600 books in total
- at least three times as many paperbacks as hardcovers

The profit on each paperback sold is £8, the profit on each hardcover sold is £20 and the profit on each deluxe edition sold is £40

Let x, y and z represent the number of paperbacks, hardcovers and deluxe editions produced.

(a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(5)

The publisher decides to solve this linear programming problem by using the two-stage simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage simplex method.

As part of your solution, you must show how

- the constraints have been made into equations by using slack variables, exactly two surplus variables and exactly two artificial variables
- the rows for the two objective functions are formed

(6)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	0	1	1	3	0	−1	−3	600
z	0	$\frac{4}{11}$	1	0	$-\frac{1}{11}$	$\frac{1}{11}$	0	$\frac{1}{11}$	$-\frac{1}{11}$	$\frac{2000}{11}$
x	1	$\frac{7}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	0	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
s_4	0	$\frac{40}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	1	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
P	0	$-\frac{4}{11}$	0	0	$-\frac{32}{11}$	$-\frac{56}{11}$	0	$\frac{32}{11}$	$\frac{56}{11}$	$\frac{204800}{11}$
I	0	0	0	0	0	0	0	1	1	0

(c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method to obtain a new tableau. Make your method clear by stating the row operations you use.

(5)

After three iterations of the second stage of the two-stage simplex method, the following tableau is obtained.

b.v.	x	y	z	s_1	s_2	s_3	s_4	Value
s_2	0	0	0	1	1	3	0	600
z	0	0	1	$\frac{1}{10}$	0	$\frac{1}{2}$	$-\frac{1}{10}$	100
x	1	0	0	$-\frac{3}{40}$	0	$-\frac{9}{8}$	$-\frac{7}{40}$	1125
y	0	1	0	$-\frac{1}{40}$	0	$-\frac{3}{8}$	$\frac{11}{40}$	375
P	0	0	0	$\frac{29}{10}$	0	$\frac{7}{2}$	$\frac{1}{10}$	20500

Given that the publisher produces the optimal number of each version of the book,

(d) (i) state the maximum profit the publisher can earn,

(ii) find the number of hours the binding machine will be used.

(2)

(e) Give a reason why the publisher may not earn the profit stated in (d)(i).

(1)

(Total for question = 19 marks)

(Q07 9FM0/03D, June 2023)

Q3.

A linear programming problem in x, y and z is described as follows.

Maximise $P = 3x + 2y + 2z$

subject to

$2x + 2y + z \leq 25$
$x + 4y \leq 15$
$x \geq 3$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

The big-M method is to be used to solve this linear programming problem.

(b) Define, in this context, what M represents. You must use correct mathematical language in your answer.

(1)

The initial tableau for a big-M solution to the problem is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	2	2	1	1	0	0	0	25
s_2	1	4	0	0	1	0	0	15
t_1	1	0	0	0	0	−1	1	3
P	$-(3+M)$	−2	−2	0	0	M	0	$-3M$

(c) Explain clearly how the equation represented in the b.v. t_1 row was derived.

(1)

(d) Show how the equation represented in the b.v. P row was derived.

(2)

The tableau obtained from the first iteration of the big-M method is shown below.

b.v.	x	y	z	s_1	s_2	s_3	t_1	Value
s_1	0	2	1	1	0	2	−2	19
s_2	0	4	0	0	1	1	−1	12
x	1	0	0	0	0	−1	1	3
P	0	−2	−2	0	0	−3	$3+M$	9

(e) Solve the linear programming problem, starting from this second tableau. You must

- give a detailed explanation of your method by clearly stating the row operations you use and
- state the solution by deducing the final values of P, x, y and z.

(7)

(Total for question = 12 marks)

(Q07 9FM0/3D-4D, Specimen papers)

Q4.

A maximisation linear programming problem in x, y and z is to be solved using the two-stage simplex method.

The partially completed initial tableau is shown below.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	1	2	3	1	0	0	0	0	45
a_1	3	2	0	0	−1	0	1	0	9
a_2	−1	0	4	0	0	−1	0	1	4
P	−2	−1	−3	0	0	0	0	0	0
A									

(a) Using the information in the above tableau, formulate the linear programming problem. State the objective and list the constraints as inequalities.

(4)

(b) Complete the bottom row of Table 1 in the answer book. You should make your method and working clear.

(2)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

Basic variable	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	0	$\frac{5}{6}$	0	1	$\frac{7}{12}$	$\frac{3}{4}$	$-\frac{7}{12}$	$-\frac{3}{4}$	$\frac{147}{4}$
x	1	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	3
z	0	$\frac{1}{6}$	1	0	$-\frac{1}{12}$	$-\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{7}{4}$
P	0	$\frac{5}{6}$	0	0	$-\frac{11}{12}$	$-\frac{3}{4}$	$\frac{11}{12}$	$\frac{3}{4}$	$\frac{45}{4}$
A	0	0	0	0	0	0	1	1	0

(c) (i) Explain how the above tableau shows that a basic feasible solution has been found for the original linear programming problem.

(ii) Write down the basic feasible solution for the second stage.

(3)

(d) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method, to obtain a new tableau, T. Make your method clear by stating the row operations you use.

(5)

(e) (i) Explain, using T, whether or not an optimal solution to the original linear programming problem has been found.

(ii) Write down the value of the objective function.

(iii) State the values of the basic variables.

(3)

(Total for question = 17 marks)

(Q07 9FM0/03D, Oct 2020)

Q5.

A linear programming problem in x, y and z is described as follows.

Maximise $P = 2x + 2y - z$

subject to $3x + y + 2z \leq 30$

$x - y + z \geq 8$

$4y + 2z \geq 15$

$x, y, z \geq 0$

(a) Explain why the Simplex algorithm cannot be used to solve this linear programming problem.

(1)

(b) Set up the initial tableau for solving this linear programming problem using the big-M method.

(7)

After a first iteration of the big-M method, the tableau is

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	3	0	1.5	1	0	0.25	0	−0.25	26.25
a_1	1	0	1.5	0	−1	−0.25	1	0.25	11.75
y	0	1	0.5	0	0	−0.25	0	0.25	3.75
P	$-(2 + M)$	0	$2 - 1.5M$	0	M	$-0.5 + 0.25M$	0	$0.5 + 0.75M$	$7.5 - 11.75M$

(c) State the value of each variable after the first iteration.

(1)

(d) Explain why the solution given by the first iteration is not feasible.

(1)

Taking the most negative entry in the profit row to indicate the pivot column,

(e) obtain the most efficient pivot for a second iteration. You must give reasons for your answer.

(2)

(Total for question = 12 marks)

(Q06 9FM0/03D, June 2019)

Q6.

Two friends, Anaira and Tommi, play a game involving two positive numbers x and y

Anaira gives Tommi the following clues to see if he can correctly determine the value of x and the value of y

- x is greater than y and the difference between the two is at least 100
- x is at most 5 times as large as y
- the sum of $2x$ and $3y$ is at least 350
- the sum of x and y is as small as possible

Tommi decides to solve this problem by using the big-M method.

(a) Set up an initial tableau for solving this problem using the big-M method.

As part of your solution, you must show

- how the constraints were made into equations using one slack variable, exactly two surplus variables and exactly two artificial variables
- how the objective function was formed

(6)

The big-M method is applied until the tableau containing the optimal solution to the problem is found. One row of this final tableau is as follows.

b.v.	x	y	s_1	s_2	s_3	a_1	a_2	Value
x	1	0	$-\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$	$\frac{1}{5}$	130

(b) (i) State the value of x

(ii) Hence deduce the value of y, making your reasoning clear.

(3)

(Total for question = 9 marks)

(Q05 9FM0/03D, June 2024)

Q7.

A linear programming problem in x, y and z is to be solved using the big-M method.
The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	−2	2	0	−1	0	1	0	8
a_2	3	0	−4	0	0	−1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for question = 6 marks)

(Q04 9FM0/03D, June 2022)