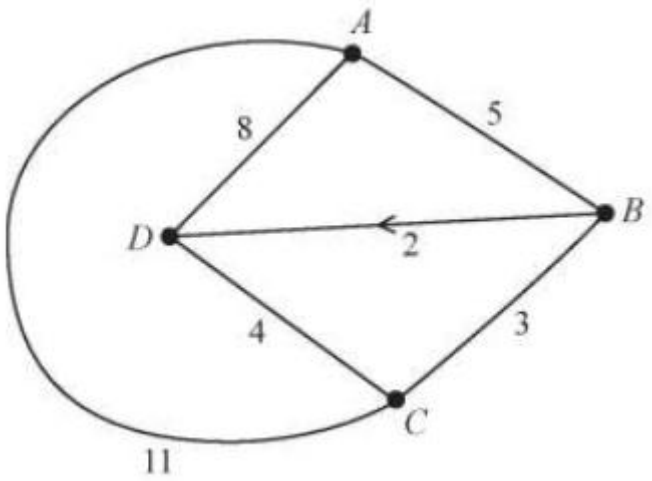
Shortest Path Exam Questions (Edexcel)

Q1.



The network in Figure 3 shows the roads linking a depot, D, and three collection points A, B and C. The number on each arc represents the length, in miles, of the corresponding road. The road from B to D is a one-way road, as indicated by the arrow.

(a) Explain clearly if Dijkstra's algorithm can be used to find a route from D to A.

(1)

The initial distance and route tables for the network are given in the answer book.

(b) Use Floyd's algorithm to find a table of least distances. You should show both the distance table and the route table after each iteration.

(7)

(c) Explain how the final route table can be used to find the shortest route from D to B. State this route.

(2)

[Note: Part (d) relates to the Travelling Salesman Problem so only attempt it if you have covered this topic.]

There are items to collect at A, B and C. A van will leave D to make these collections in any order and then return to D. A minimum route is required.

Using the final distance table and the Nearest Neighbour algorithm starting at D,

(d) find a minimum route and state its length.

(2)

Floyd's algorithm and Dijkstra's algorithm are applied to a network. Each will find the shortest distance between vertices of the network.

(e) Describe how the results of these algorithms differ.

(2)

(Total for question = 14 marks)

(Q04 9FM0/3D-4D, Specimen papers)

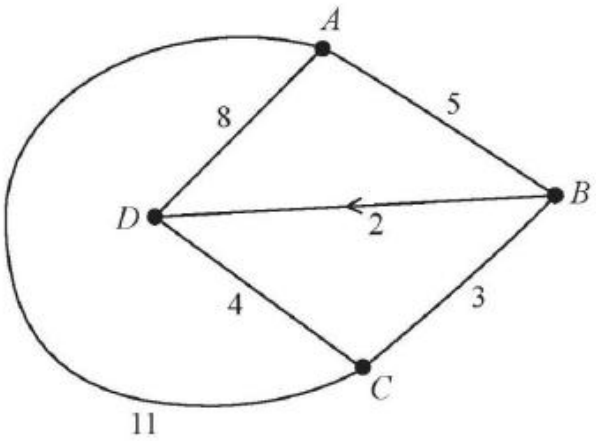


Figure 3

	A	B	C	D
A	-	5	11	8
B	5	-	3	2
C	11	3	-	4
D	8	∞	4	-

	A	B	C	D
A	A	B	C	D
B	A	B	C	D
C	A	B	C	D
D	A	B	C	D

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
B				
C				
D				

	A	B	C	D
A				
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	A	B	C	D
A				
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D				

	A	B	C	D
A				
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	A	B	C	D
A				
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Q2.

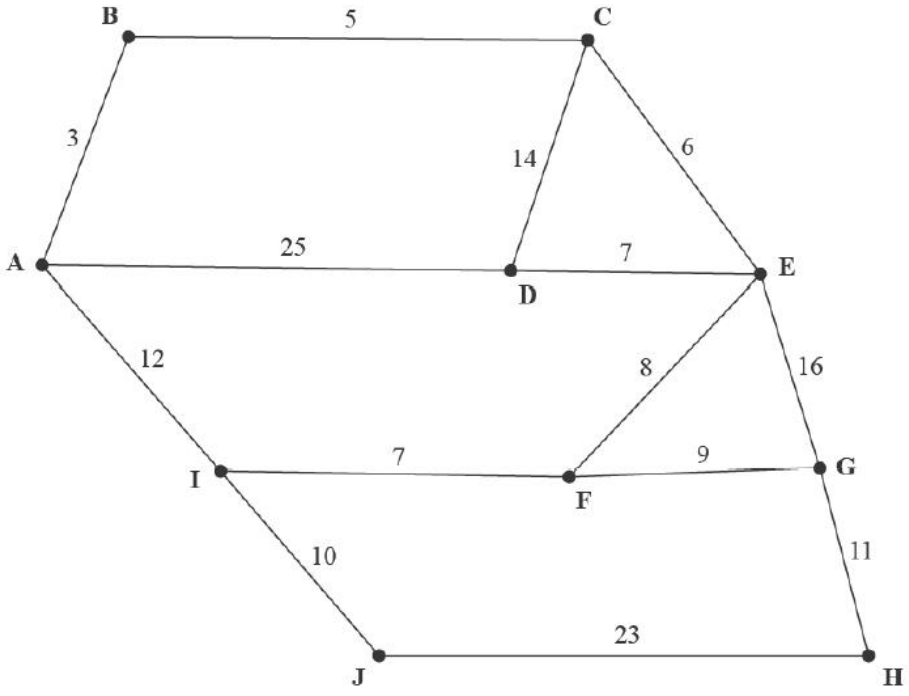


Figure 1

Figure 1 represents a network of roads.
The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest time needed to travel from A to H.

(ii) State the quickest route.

(6)

(b) If it takes 1.5 seconds to run the algorithm when $n = 250$, calculate approximately how long it will take, in seconds, to run the algorithm when $n = 9500$. You should make your method and working clear.

(2)

(c) Explain why your answer to part (b) is only an approximation.

(1)

(Total for question = 9 marks)

(Q01 8FM0/27, June 2018)

Q3.

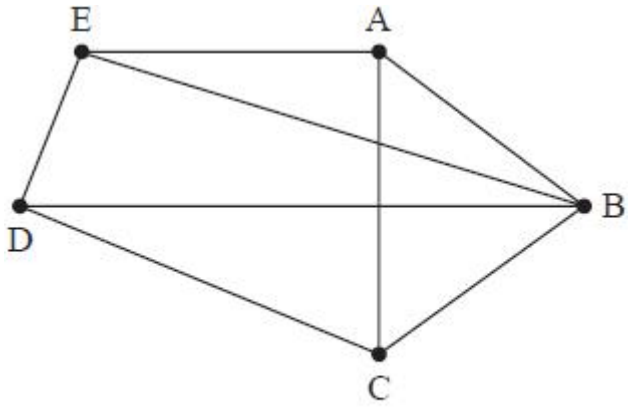


Figure 1

Figure 1 shows the graph G.

(a) State whether G is Eulerian, semi-Eulerian, or neither, giving a reason for your answer.

(1)

(b) Write down an example of a Hamiltonian cycle on G.

(1)

(c) State whether or not G is planar, justifying your answer.

(1)

(d) State the number of arcs that would need to be added to G to make the graph K_5

(1)

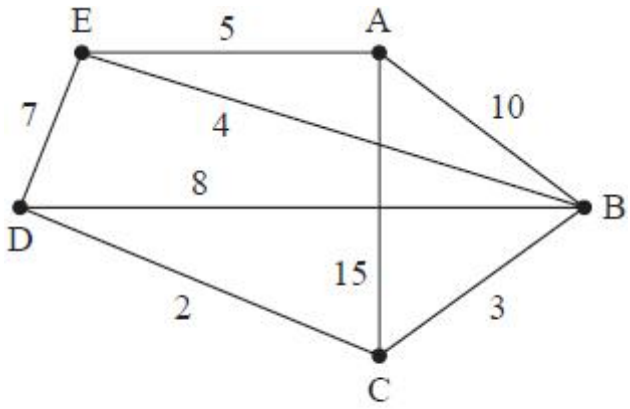


Figure 2

Direct roads between five villages, A, B, C, D and E, are represented in Figure 2. The weight on each arc is the time, in minutes, required to travel along the corresponding road. Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

(e) For the network represented in Figure 2, complete the initial time matrix in the answer book.

(1)

The time matrix after four iterations of Floyd's algorithm is shown in Table 1.

	A	B	C	D	E
A	–	10	13	15	5
B	10	–	3	5	4
C	13	3	–	2	7
D	15	5	2	–	7
E	5	4	7	7	–

Table 1

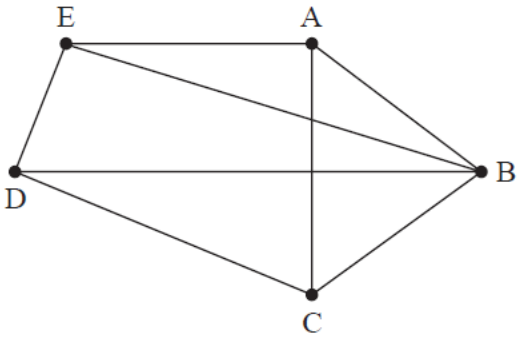
(f) Perform the final iteration of Floyd's algorithm that follows from Table 1, showing the time matrix for this iteration.

(2)

(Total for question = 7 marks)

(Q01 9FM0/03D, June 2023)

(e) Initial time matrix



	A	B	C	D	E
A	–				
B		–			
C			–		
D				–	
E					–

(f) Final iteration

	A	B	C	D	E
A	–				
B		–			
C			–		
D				–	
E					–

Q4.

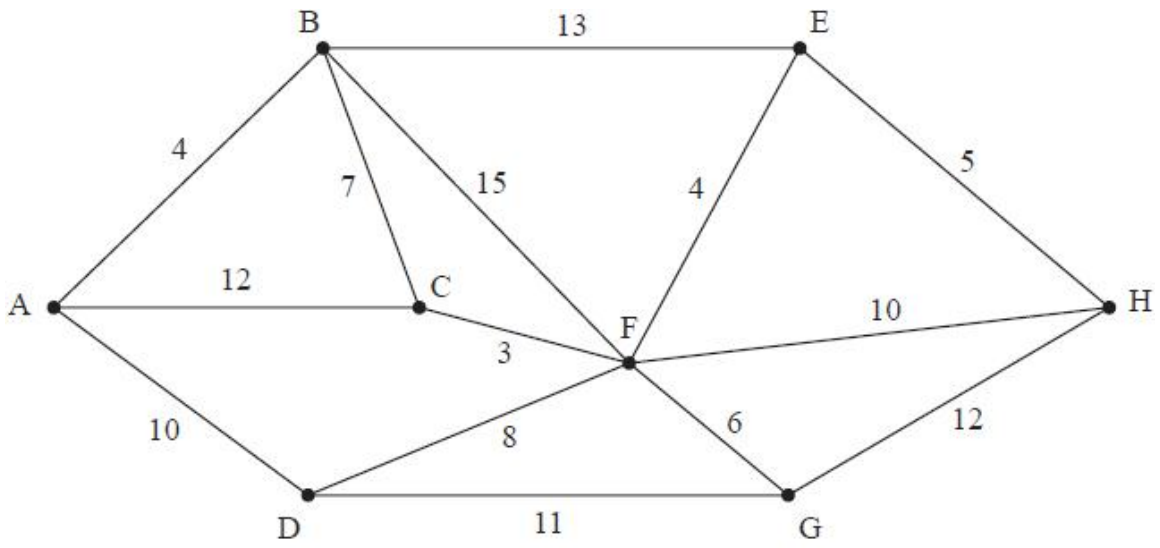


Figure 2

[*The total weight of the network is* 120]

(a) Explain what is meant by the term "path".

(2)

(b) State, with a reason, whether the network in Figure 2 is Eulerian, semi-Eulerian or neither.

(1)

Figure 2 represents a network of cycle tracks between eight villages, A, B, C, D, E, F, G and H. The number on each arc represents the length, in km, of the corresponding track. Samira lives in village A, and wishes to visit her friend, Daisy, who lives in village H.

(c) Use Dijkstra's algorithm to find the shortest path that Samira can take.

(5)

[Part (d) is on the topic of Route Inspection so only attempt it if you have covered this topic]

An extra cycle track of length 9 km is to be added to the network. It will either go directly between C and D or directly between E and G.

Daisy plans to cycle along every track in the new network, starting and finishing at H.

Given that the addition of either track CD or track EG will not affect the final values obtained in (c),

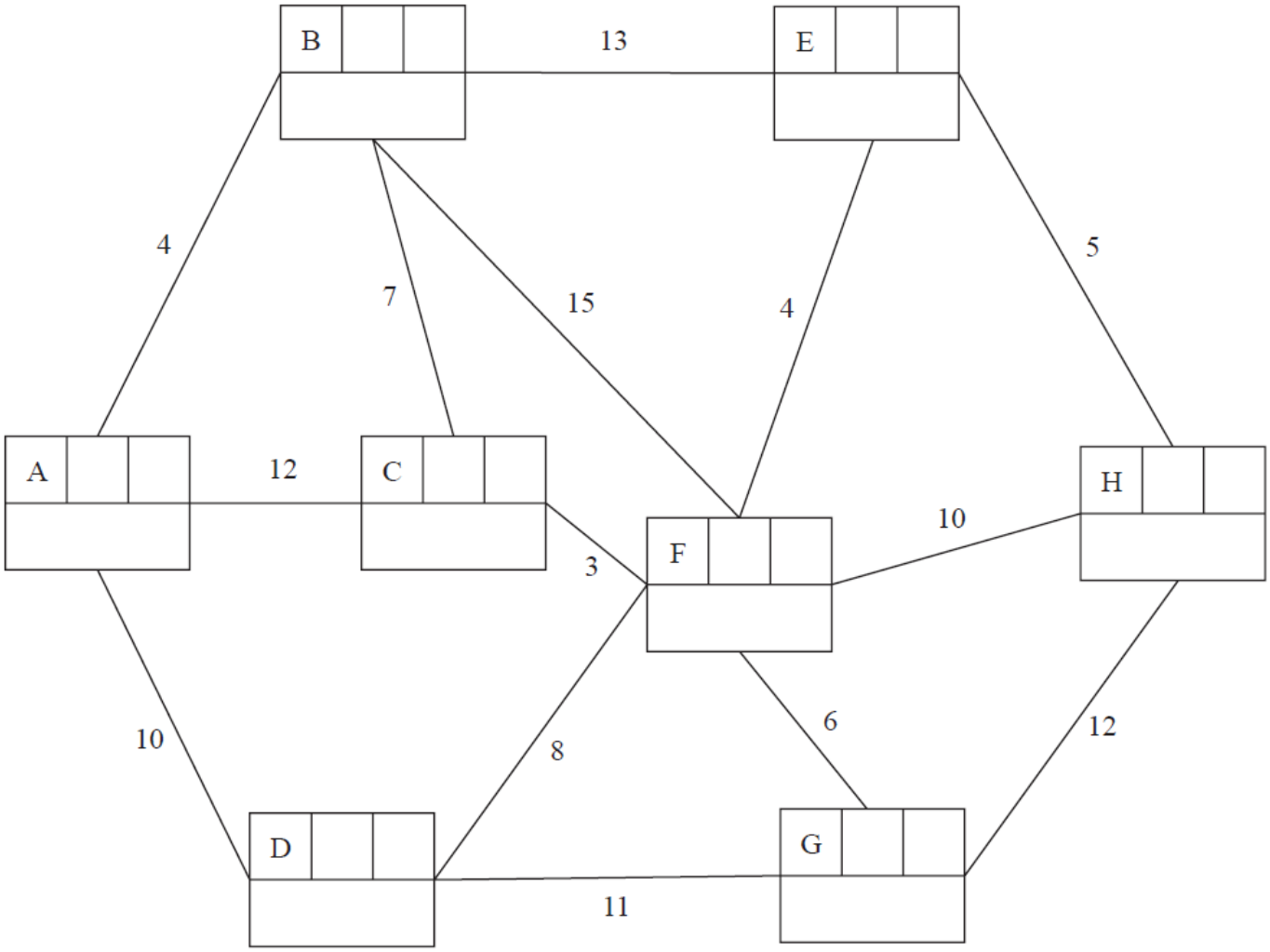
(d) use a suitable algorithm to find out which of the two possible extra tracks will give Daisy the shortest route, making your method and working clear. You must

- state which tracks Daisy will repeat in her route
- state the total length of her route

(6)

(Total for question = 14 marks)

(Q03 8FM0/27, June 2022)



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path _____

Q5.

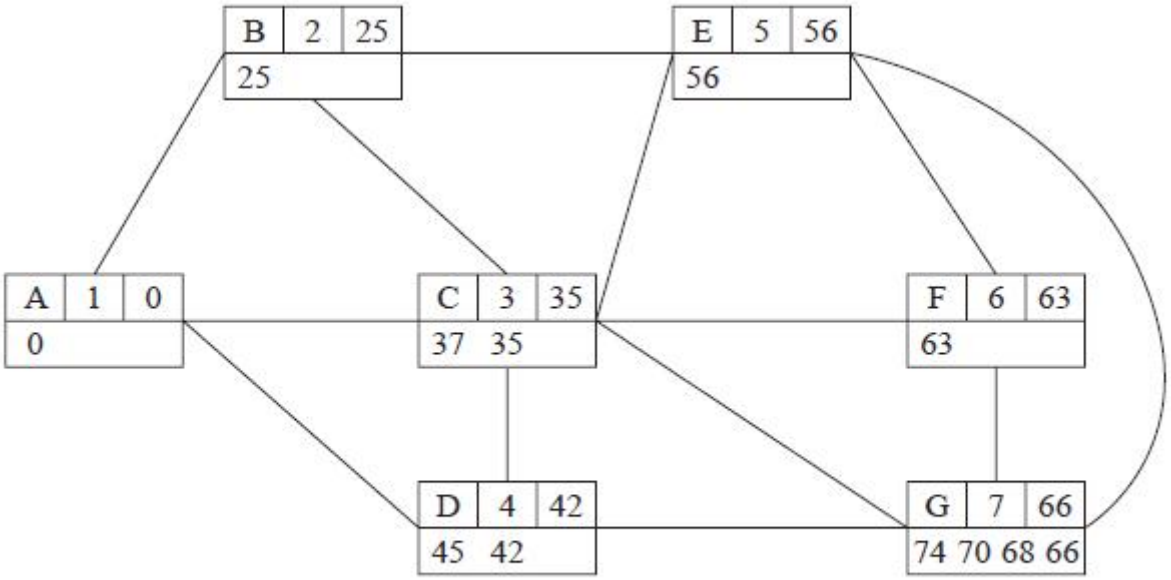


Figure 2

Dijkstra's algorithm has been applied to the network in Figure 2.

A working value has only been replaced at a node if the new working value is smaller.

(a) State the length of the shortest path from A to G.

(1)

(b) Complete the table in the answer book giving the weight of each arc listed.
(Note that arc CE and arc EF are not in the table.)

(3)

(c) State the shortest path from A to G.

(1)

It is now given that

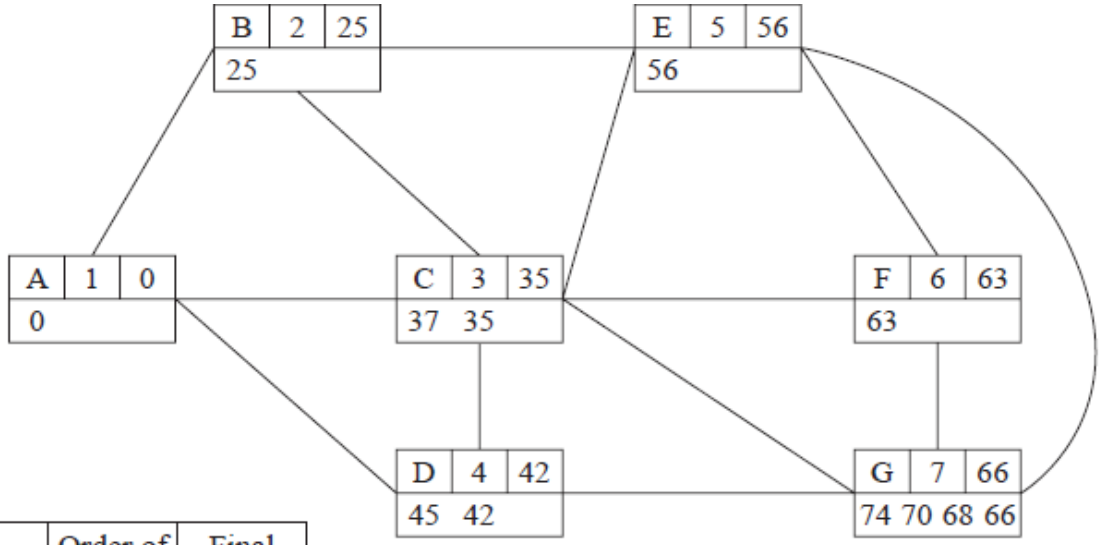
- when Prim's algorithm, starting from A, is applied to the network, the order in which the arcs are added to the tree is AB, BC, CD, CE, EF and FG
- the weight of the corresponding minimum spanning tree is 80
- the shortest path from A to F via E has weight 67

(d) Determine the weight of arc CE and the weight of arc EF, making your reasoning clear.

(3)

(Total for question = 8 marks)

(Q04 8FM0/27, Oct 2021)



Key:

Vertex	Order of labelling	Final value
Working values		

(a) length of the shortest path from A to G: _____

_____

(b)

Arc	Weight
AB	
AC	
AD	
BC	
BE	
CD	
CF	
CG	
DG	
EG	
FG	

Q6.

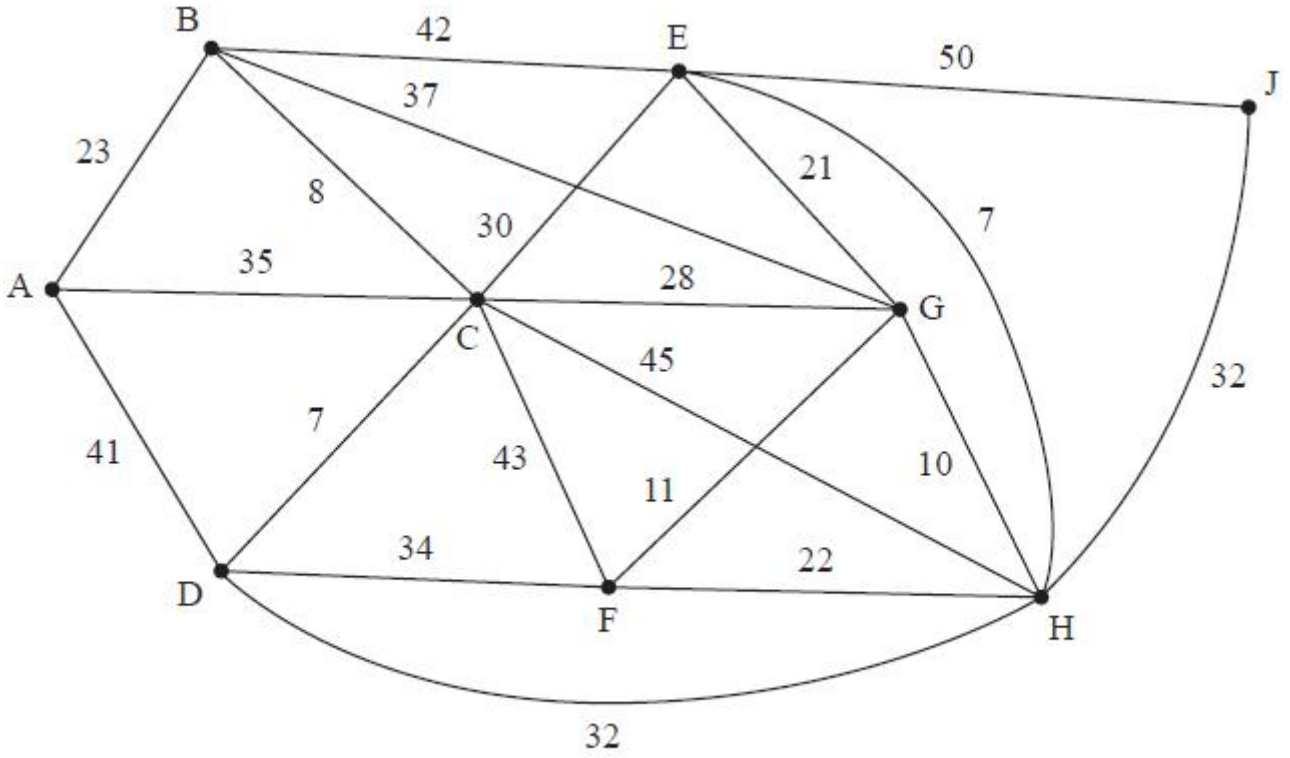


Figure 4

Figure 4 represents a network with nodes, A, B, C, D, E, F, G, H and J.
The number on each edge gives the length of the corresponding edge.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

One application of Dijkstra's algorithm has order n^2, where n is the number of nodes in the network.

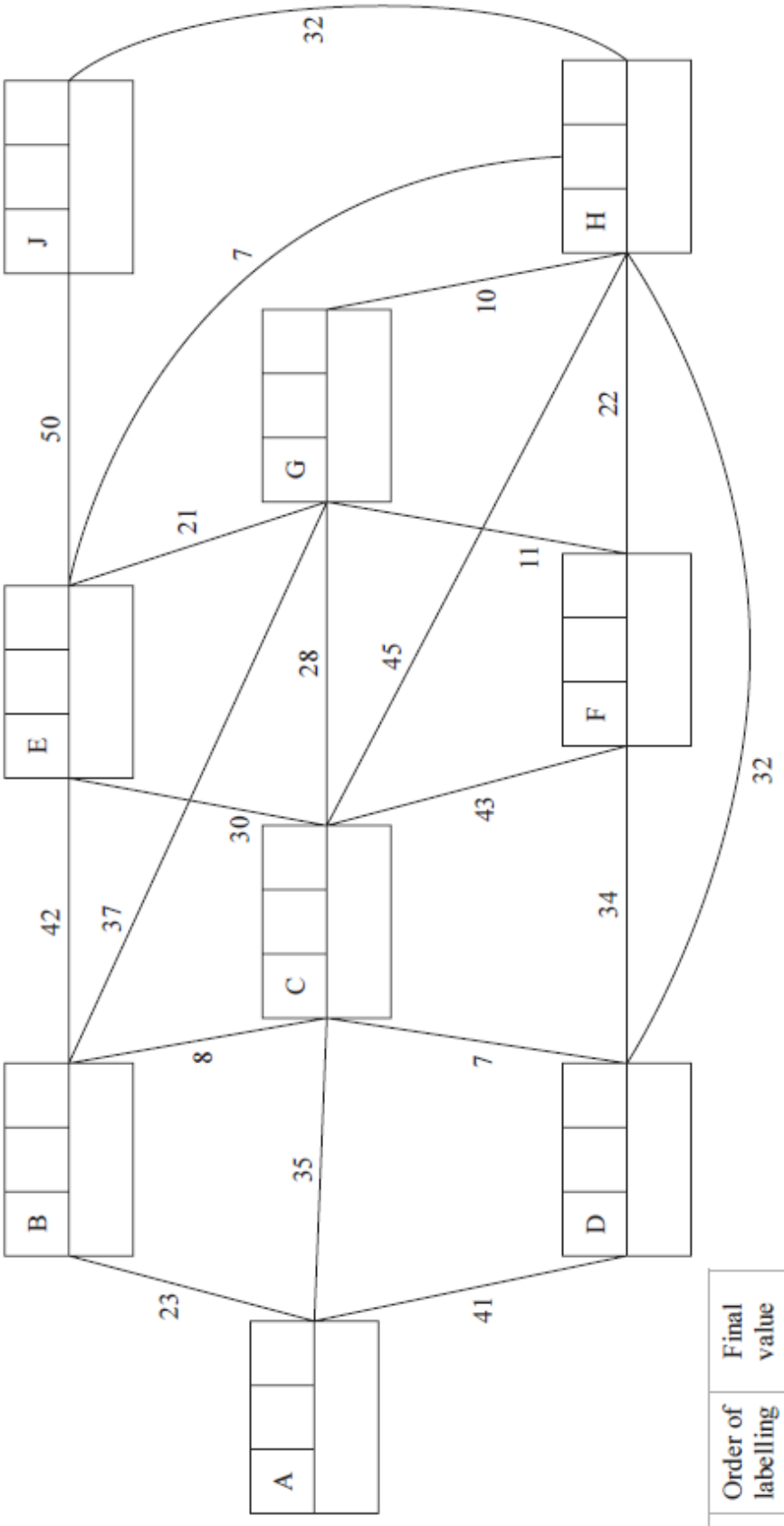
It takes a computer 0.0312 seconds to find the shortest path from a given start node to a given end node in a network of 9 nodes.

(b) Calculate approximately how long it would take, in minutes, for the computer to find the shortest path from a given start node to a given end node for a network of 9000 nodes.

(2)

(Total for question = 8 marks)

(Q03 9FM0/03D, June 2023)



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to J: _____

Length of shortest path from A to J: _____

Q7.

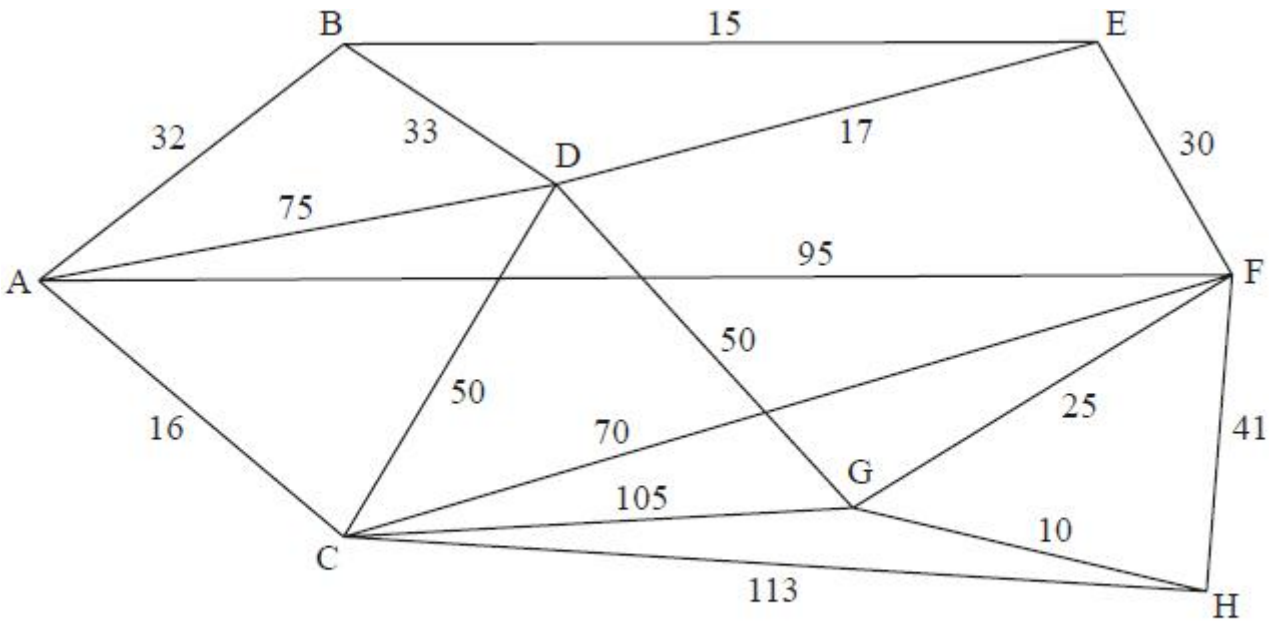


Figure 4

In Figure 4 the weights on the arcs represent distances.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to H.

(ii) State the length of the shortest path from A to H.

(6)

One application of Dijkstra's algorithm has order n^2, where n is the number of nodes in the network. A computer produces a table of shortest distances between any two different nodes by repeatedly applying Dijkstra's algorithm from each node of the network.

It takes the computer 0.082 seconds to produce a table of shortest distances for a network of 10 nodes.

(b) Calculate approximately how long it will take, in seconds, for the computer to produce a table of shortest distances for a network with 200 nodes. You must give a reason for your answer.

(3)

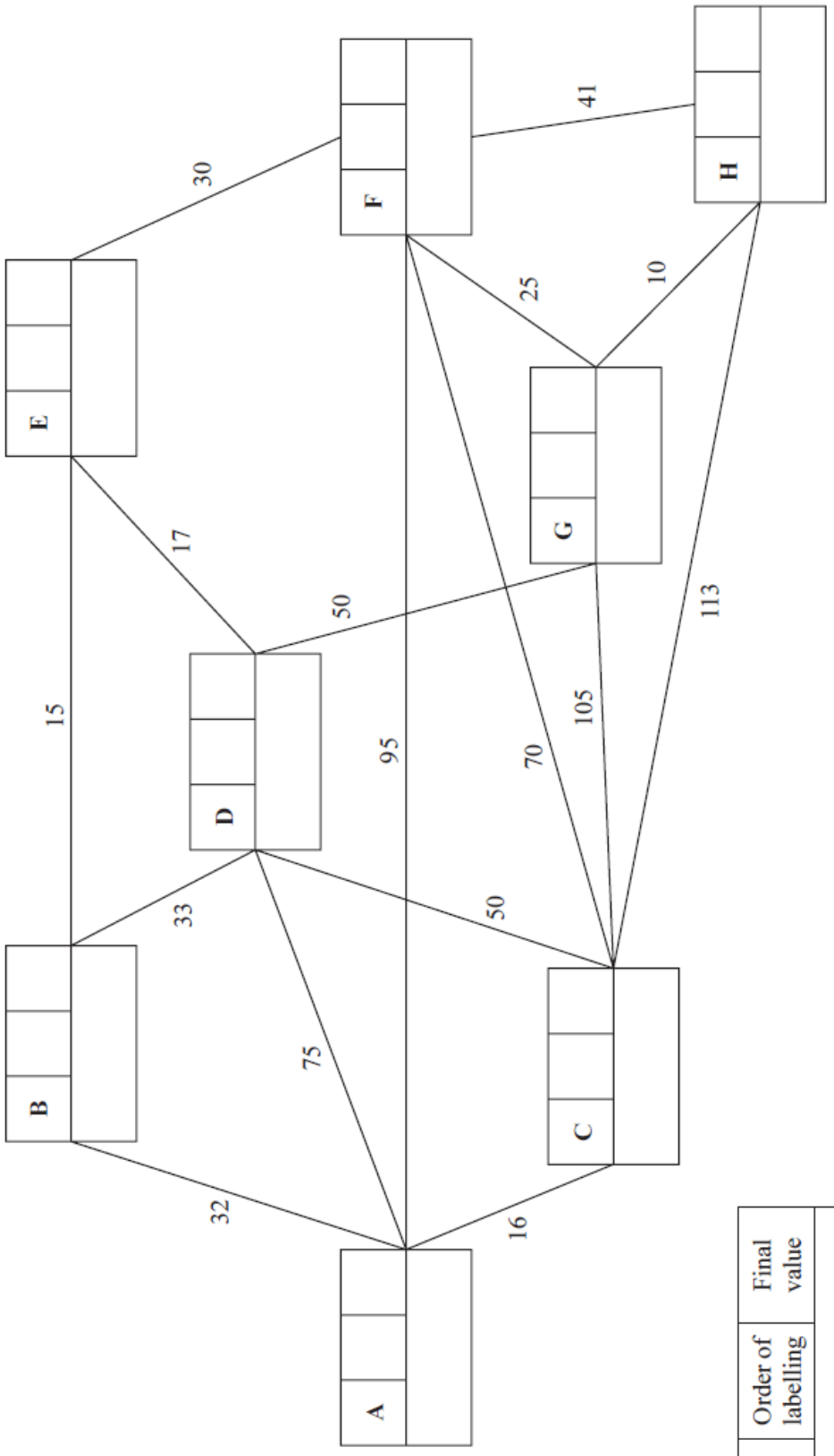
(c) Explain why your answer to part (b) can only be an approximation.

(1)

(Total for question = 10 marks)

(Q06 9FM0/03D, Oct 2021)

6.



Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path from A to H: _____

Length of shortest path from A to H: _____