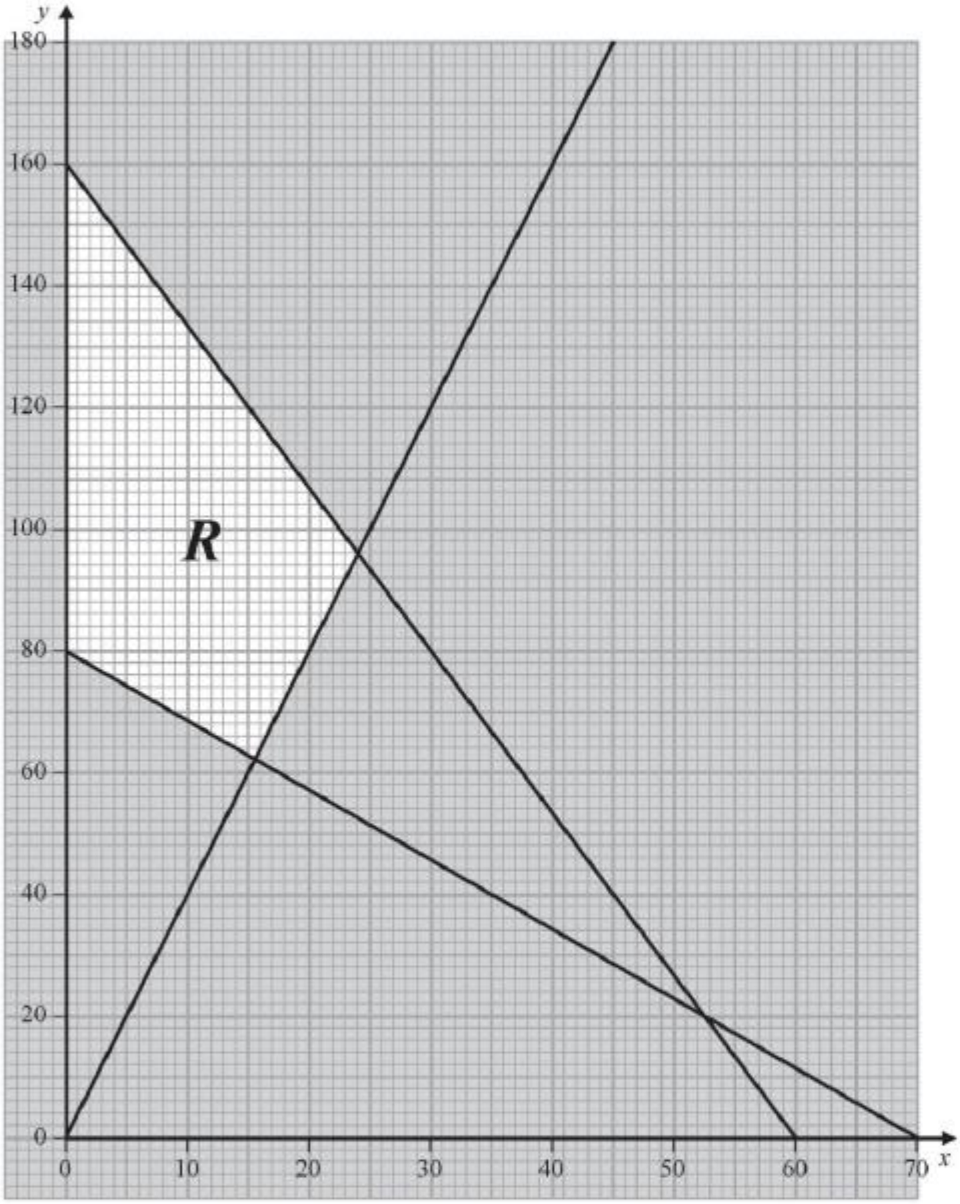
Year 1 D1 – Linear Programming – Graphical Method - Exam Questions (Edexcel)

Q1.



A teacher buys pens and pencils. The number of pens, x, and the number of pencils, y, that he buys can be represented by a linear programming problem as shown in Figure 2, which models the following constraints:

$$8x + 3y \leq 480$$

$$8x + 7y \geq 560$$

$$y \geq 4x$$

$$x, y \geq 0$$

The total cost, in pence, of buying the pens and pencils is given by

$$C = 12x + 15y$$

Determine the number of pens and the number of pencils which should be bought in order to minimise the total cost. You should make your method and working clear.

(Total for question = 7 marks)

(Q02 8FM0/2K/sA, Specimen papers)

Q2.

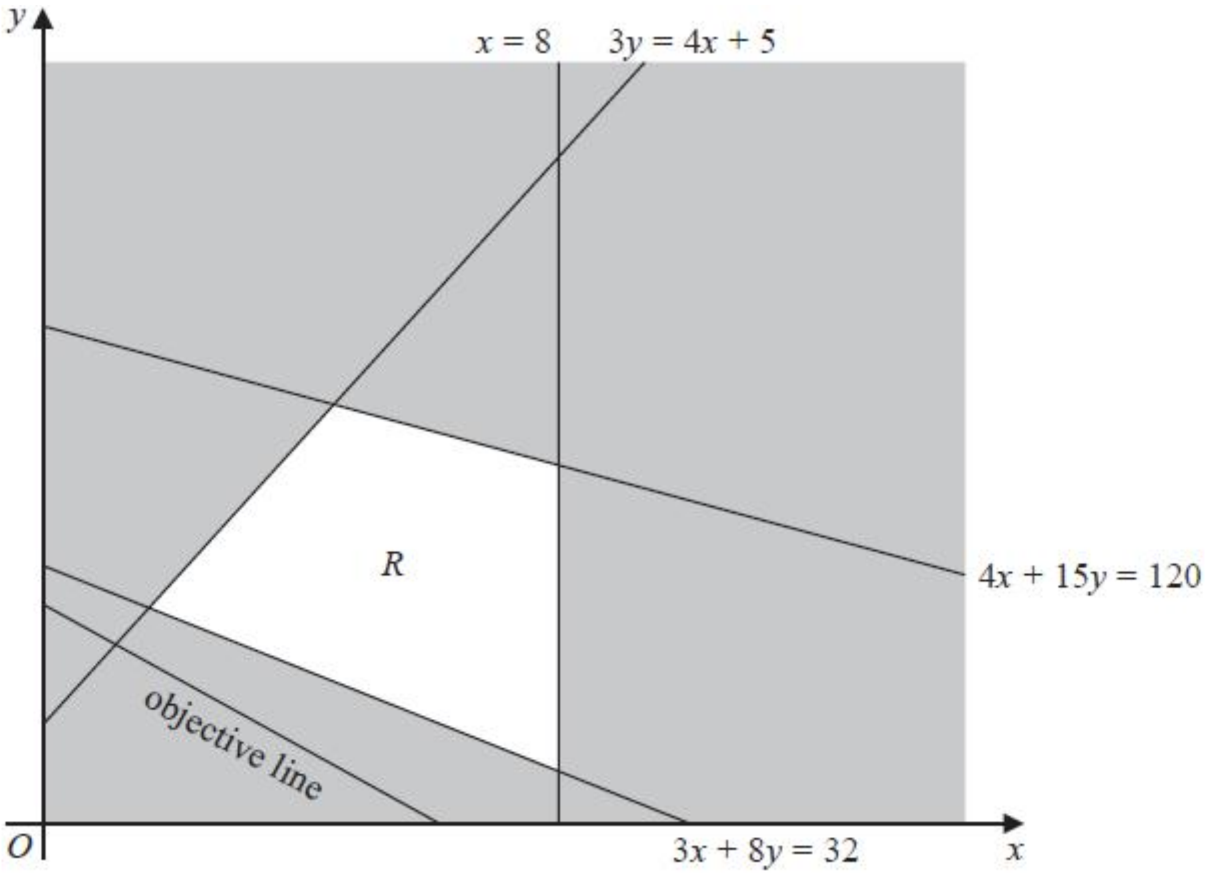


Figure 3

Figure 3 shows the constraints of a linear programming problem in x and y.

The unshaded area, including its boundaries, forms the feasible region, R.

An objective line has been drawn and labelled on the graph.

(a) State the inequalities that define the feasible region.

(2)

The maximum value of the objective function is $\frac{160}{3}$

The minimum value of the objective function is $\frac{883}{41}$

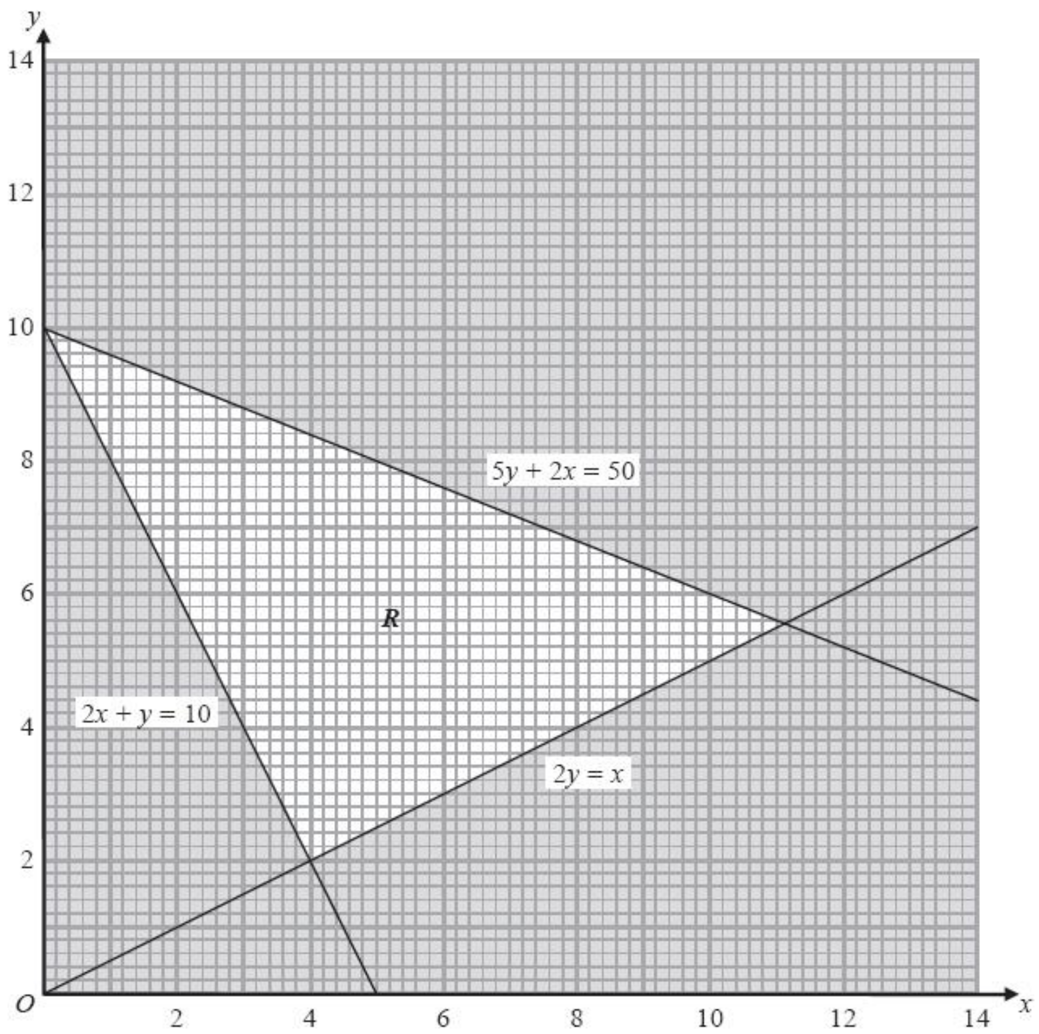
(b) Determine the objective function, showing your working clearly.

(5)

(Total for question = 7 marks)

(Q04 8FM0/27, June 2023)

Q3.



The diagram shows the constraints of a linear programming problem in x and y, where R is the feasible region.

(a) Write down the inequalities that form region R.

(2)

(b) Find the exact coordinates of the vertices of the feasible region.

(3)

The objective is to maximise P, where $P = 2x + 3y$

(c) Use point testing to find the optimal vertex, V, of the feasible region.

(2)

The objective is changed to maximise Q, where $Q = 2x + \lambda y$

Given that λ is a constant and V is still the only optimal vertex of the feasible region,

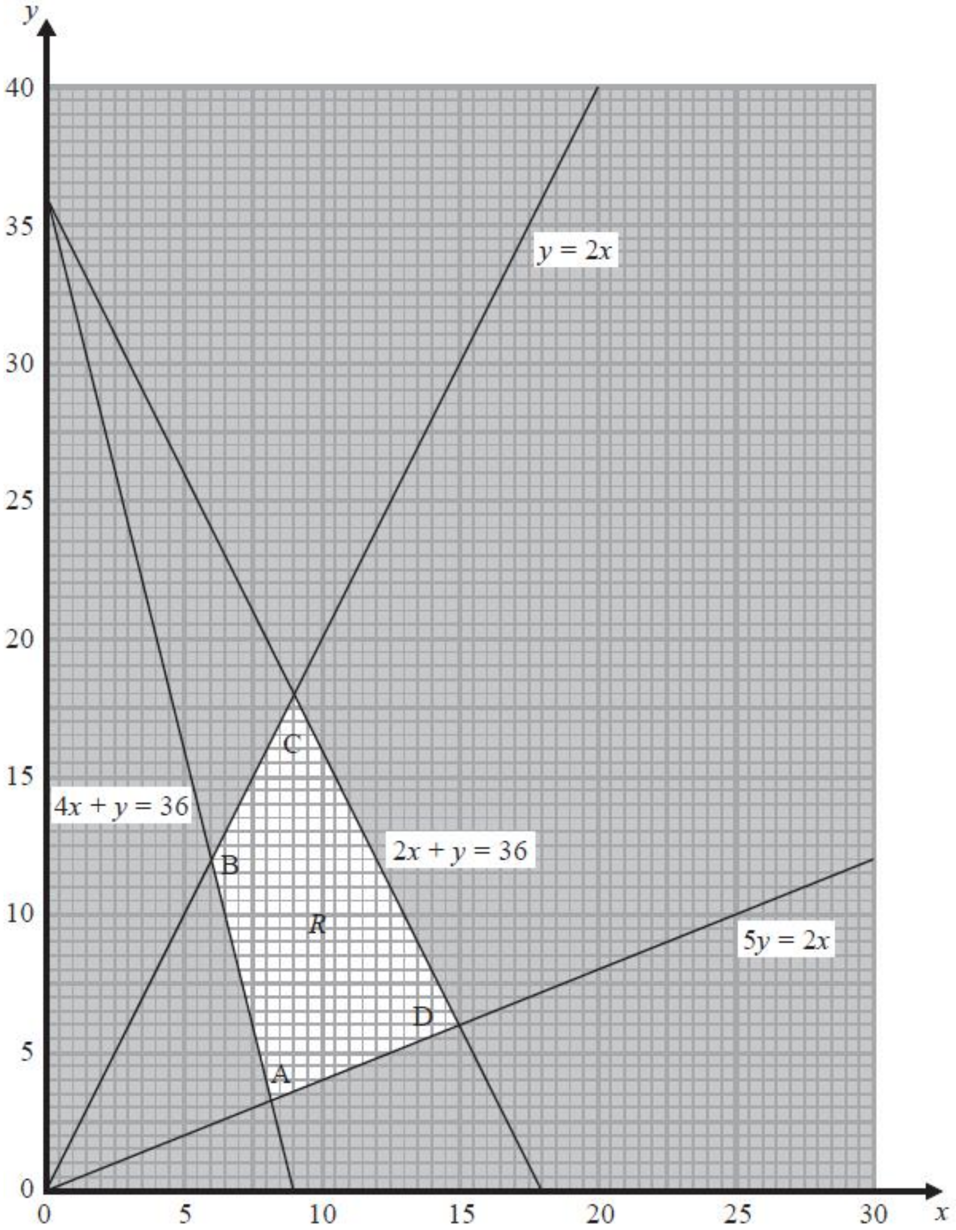
(d) find the range of possible values of λ.

(4)

(Total for question = 11 marks)

(Q04 6689/01, June 2017)

Q4.



The graph is being used to solve a linear programming problem. The four constraints have been drawn on the graph and the rejected regions have been shaded out. The four vertices of the feasible region R are labelled A, B, C and D.

(a) Write down the constraints represented on the graph.

(2)

The objective function, P, is given by

$$P = x + ky$$

where k is a positive constant.

The minimum value of the function P is given by the coordinates of vertex A **and** the maximum value of the function P is given by the coordinates of vertex D.

(b) Find the range of possible values for k. You must make your method clear.

(6)

(Total 8 marks)

(Q06 6689/01, June 2014)

Q5.

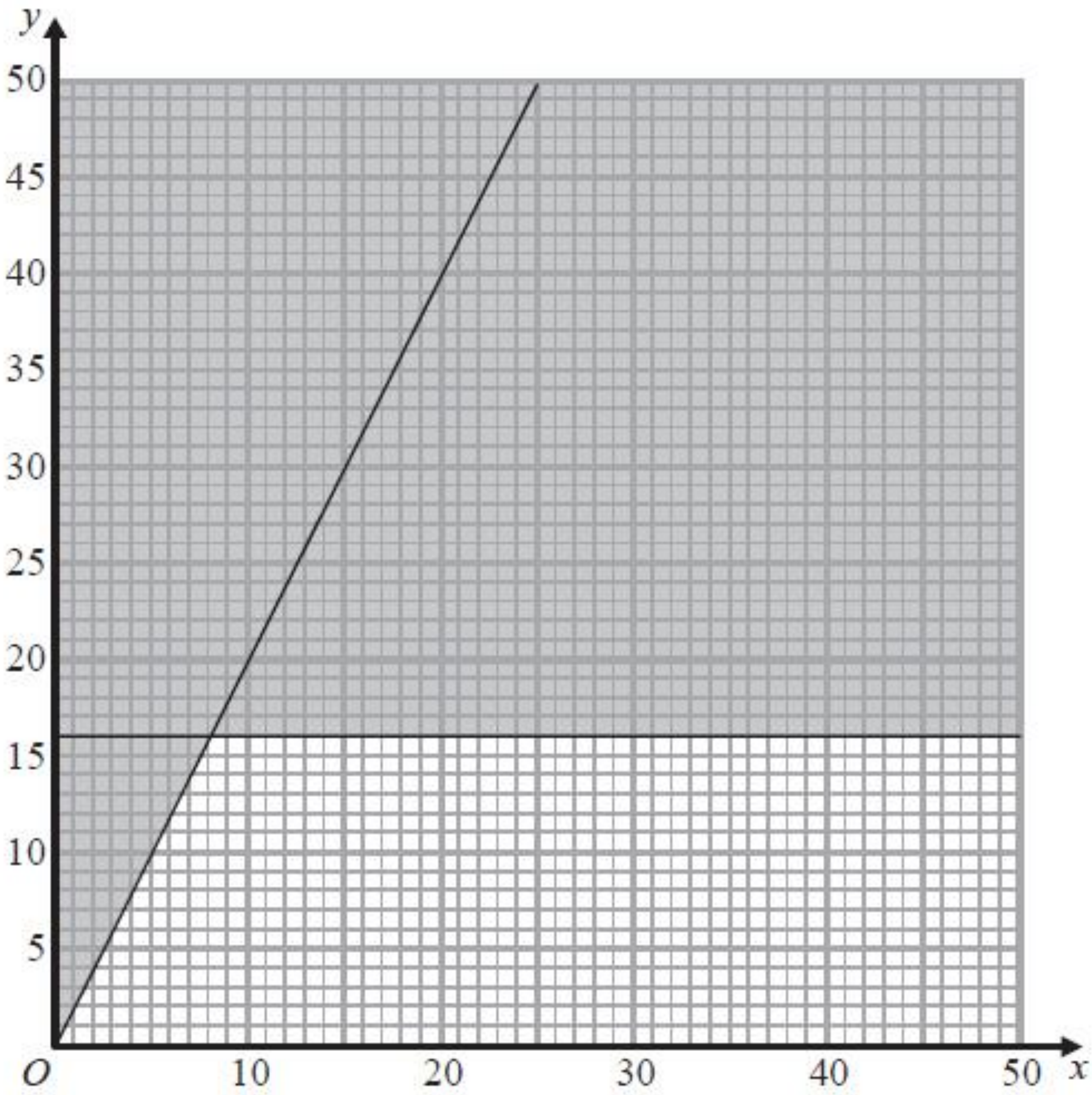


Figure 6

.A company makes two types of garden bench, the 'Rustic' and the 'Contemporary'. The company wishes to maximise its profit and decides to use linear programming.

Let x be the number of 'Rustic' benches made each week and y be the number of 'Contemporary' benches made each week.

The graph in Figure 6 is being used to solve this linear programming problem.

Two of the constraints have been drawn on the graph and the rejected region shaded out.

(a) Write down the constraints shown on the graph giving your answers as inequalities in terms of x and y.

(3)

It takes 4 working hours to make one 'Rustic' bench and 3 working hours to make one 'Contemporary' bench. There are 120 working hours available in each week.

(b) Write down an inequality to represent this information.

(2)

Market research shows that 'Rustic' benches should be at most $\frac{3}{4}$ of the total benches made each week.

(c) Write down, and simplify, an inequality to represent this information. Your inequality must have integer coefficients.

(2)

(d) Add two lines and shading to Figure 6 in your answer book to represent the inequalities of (b) and (c). Hence determine and label the feasible region, R.

(3)

The profit on each 'Rustic' bench and each 'Contemporary' bench is £45 and £30 respectively.

(e) Write down the objective function, P, in terms of x and y.

(1)

(f) Determine the coordinates of each of the vertices of the feasible region and hence use the vertex method to determine the optimal point.

(4)

(g) State the maximum weekly profit the company could make.

(1)

(Total 16 marks)

(Q05 6689/01/R, June 2013)

Q6.

Charlie needs to buy storage containers.

There are two different types of storage container available, standard and deluxe.

Standard containers cost £20 and deluxe containers cost £65. Let x be the number of standard containers and y be the number of deluxe containers.

The maximum budget available is £520

(a) Write down an inequality, in terms of x and y, to model this constraint.

(1)

Three further constraints are:

$$x \geqslant 2$$
$$-x + 24y \geqslant 24$$
$$7x + 8y \leqslant 112$$

(b) Add lines and shading to Diagram 1 in the answer book to represent all four constraints.

Hence determine the feasible region and label it R.

(4)

The capacity of a deluxe container is 50% greater than the capacity of a standard container.
Charlie wishes to maximise the total capacity.

(c) State an objective function, in terms of x and y.

(1)

(d) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex V.

(3)

(e) Calculate the exact coordinates of vertex V.

(2)

(f) Determine the number of each type of container that Charlie should buy. You must make your method clear and calculate the cost of purchasing the storage containers.

(3)

(Total for question = 14 marks)

(Q06 6689/01, June 2016)

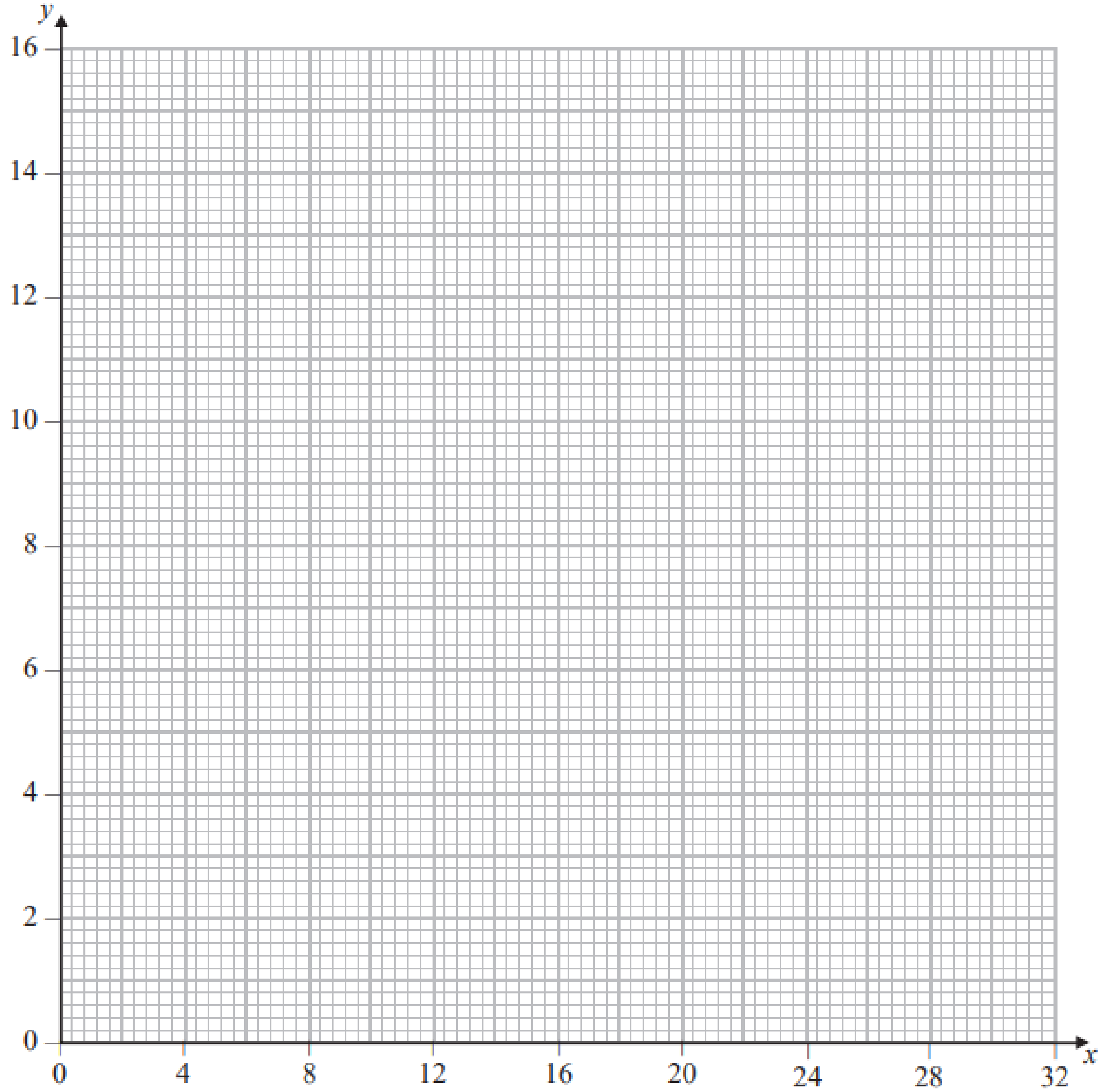


Diagram 1

Q7.

A linear programming problem in x and y is described as follows.

Minimise $C = 2x + 3y$

subject to

$x + y \geqslant 8$
$x < 8$
$4y \geqslant x$
$3y \leqslant 9 + 2x$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints.

(4)

(b) Hence determine the feasible region and label it R.

(1)

(c) Use the objective line (ruler) method to find the exact coordinates of the optimal vertex, V, of the feasible region. You must draw and label your objective line clearly.

(3)

(d) Calculate the corresponding value of C at V.

(1)

The objective is now to maximise $2x + 3y$, where x and y are integers.

(e) Write down the optimal values of x and y and the corresponding maximum value of $2x + 3y$.

(2)

A further constraint, $y \leqslant kx$, where k is a positive constant, is added to the linear programming problem

(f) Determine the least value of k for which this additional constraint does not affect the feasible region.

(2)

(Total for question = 13 marks)

(Q04 6689/01, June 2015)

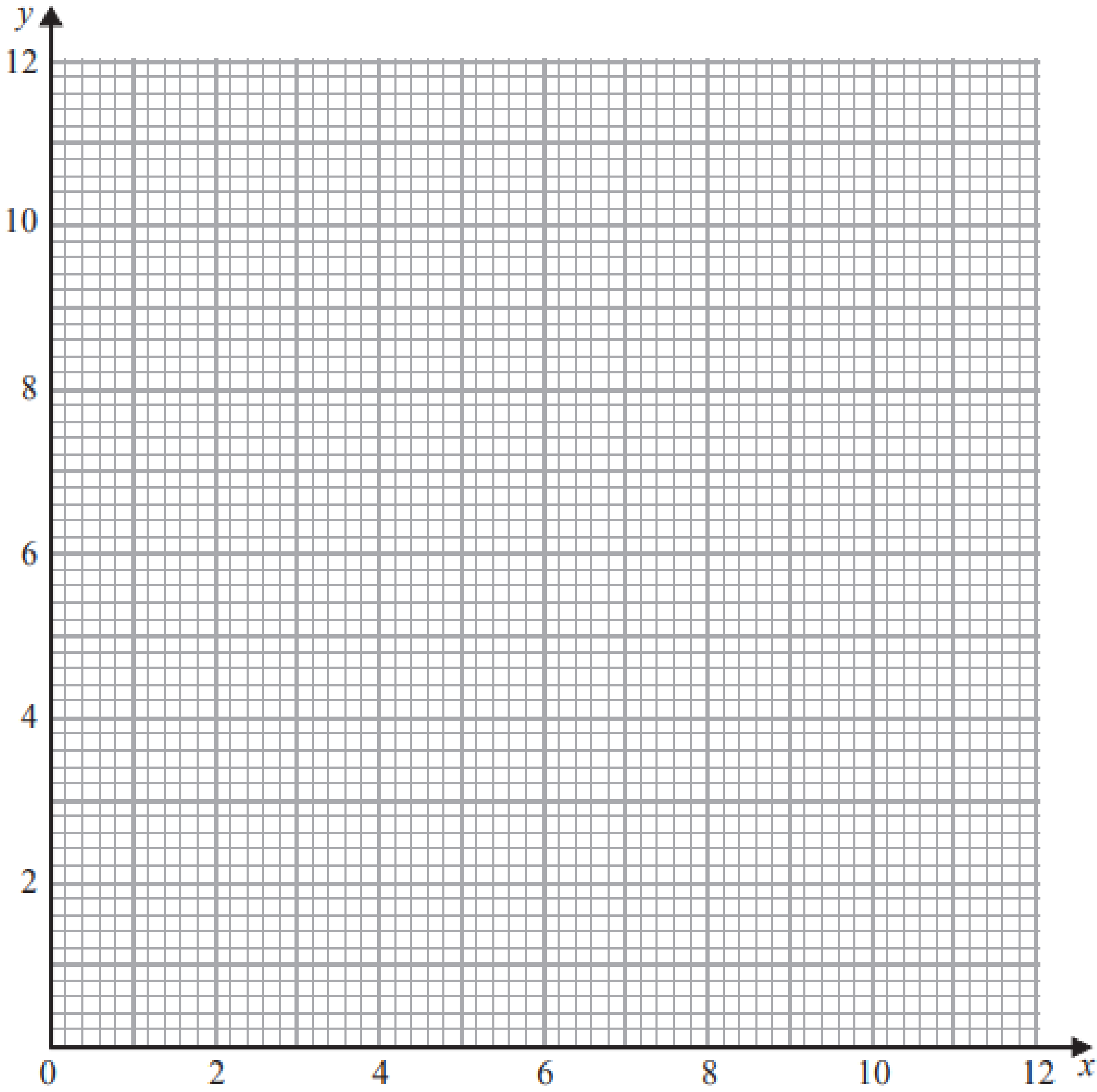


Diagram 1