



D1 (Year 1) Graph Theory Exam Questions (Edexcel)

Q1.

- (a) Draw the graph K_5 (1)
- (b) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.
 (ii) Draw two semi-Eulerian subgraphs of K_5 , each having five vertices but with a different number of edges. (3)
- (c) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree. (2)

(Total for question = 6 marks)
(Q01 8FM0/27, June 2019)

Q2.

- (a) Explain why it is **not** possible to draw a graph with exactly six nodes with degrees 1, 2, 3, 4, 5 and 6 (1)

A tree, T , has exactly six nodes. The degrees of the six nodes of T are

$$1 \quad 2 \quad (4 - x) \quad (2x - 5) \quad (4x - 11) \quad (3x - 5)$$

where x is an integer.

- b) Explain how you know that T cannot be Eulerian. (1)
- (c) (i) Determine the value of x
 (ii) Hence state whether T is semi-Eulerian or not. You must justify your answer. (5)

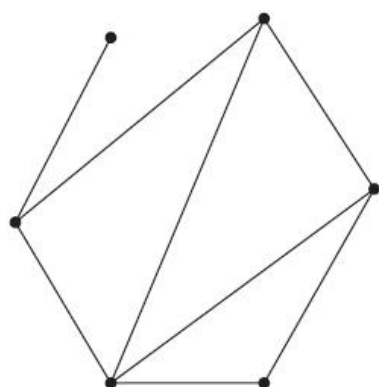


Figure 2

Figure 2 shows a graph, G , with six nodes with degrees 1, 2, 3, 3, 3 and 4

- (d) Using the vertices in Diagram 1 in the answer book, draw a graph with exactly six nodes with degrees 1, 2, 3, 3, 3 and 4 that is **not** isomorphic to G . (1)

(Total for question = 8 marks)
(Q04 9FM0/03D, June 2024)



- Q3.**
 (a) Explain why it is not possible to draw a graph with exactly 5 nodes with orders 1, 3, 4, 4 and 5 (1)

A connected graph has exactly 5 nodes and contains 18 arcs.
 The orders of the 5 nodes are $2^{2x} - 1$, 2^x , $x + 1$, $2^{x+1} - 3$ and $11 - x$

- (b) (i) Calculate x .
 (ii) State whether the graph is Eulerian, semi-Eulerian or neither. You must justify your answer. (6)

(c) Draw a graph which satisfies all of the following conditions:

- The graph has exactly 5 nodes.
- The nodes have orders 2, 2, 4, 4 and 4
- The graph is not Eulerian.

(2)

(Total for question = 9 marks)

(Q04 8FM0/2K/sA, Specimen papers)

- Q4.** [The weight of the network is $20x + 3$]
 Figure 4 shows a graph G that contains 8 arcs and 6 vertices.
 (a) State the minimum number of arcs that would need to be added to make G into an Eulerian graph. (1)
 (b) Explain whether or not the route $A - C - F - E - C - D - B$ is an example of a path on G . (1)

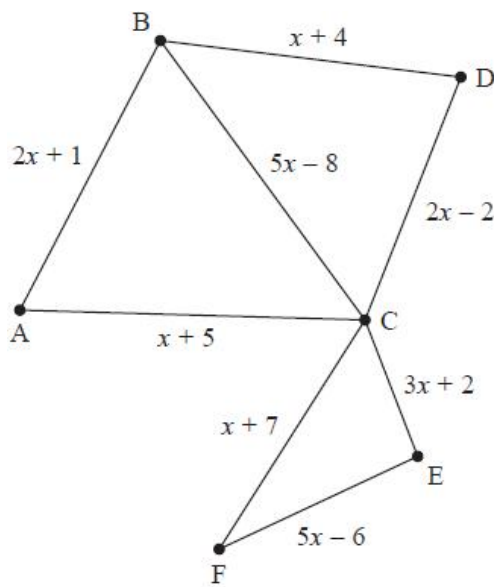


Figure 4

(Total for question = 2 marks)

(Q05 8FM0/27, June 2023)



Q5.

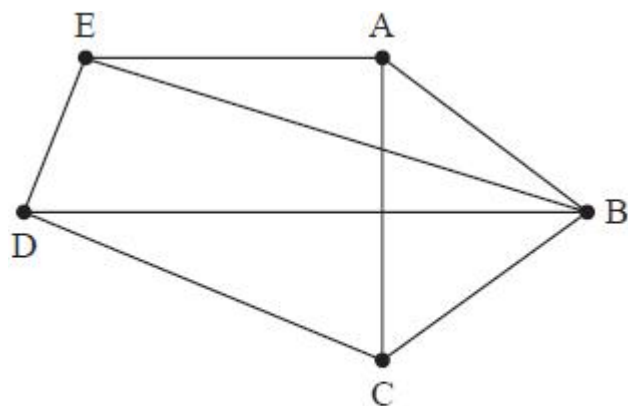


Figure 1

Figure 1 shows the graph G.

(a) State whether G is Eulerian, semi-Eulerian, or neither, giving a reason for your answer.

(1)

(b) Write down an example of a Hamiltonian cycle on G.

(1)

[Note: Part (c) is not examinable at AS Level]

(c) State whether or not G is planar, justifying your answer.

(1)

(d) State the number of arcs that would need to be added to G to make the graph K_5

(1)

(Total for question = 4 marks)

(Q01 9FM0/03D, June 2023)

[Note: not many questions on this topic have come up on the current specification so the following questions are from a different specification (OCR Legacy) which look to match the style and difficulty of what is required]

Q6.

(i) Draw a graph with five vertices of orders 1, 2, 2, 3 and 4. [2]

(ii) State whether the graph from part (i) is Eulerian, semi-Eulerian or neither. Explain how you know which it is. [2]

(iii) Explain why a graph with five vertices of orders 1, 2, 2, 3 and 4 cannot be a tree. [2]

[Total: 6 Marks]
[OCR A, D1, Jan 2009, Q2]

Q7.

(i) A graph has six vertices; two are of order 3 and the rest are of order 4. Calculate the number of arcs in the graph, showing your working. [2]

(ii) Is the graph Eulerian, semi-Eulerian or neither? Give a reason to support your answer. [1]

[Total: 3 Marks]
[OCR A, D1, Jan 2005, Q2]



Q8.

A puzzle involves a 3 by 3 grid of squares, numbered 1 to 9, as shown in Fig. 1a below. Eight of the squares are covered by blank tiles. Fig. 1b shows the puzzle with all of the squares covered except for square 4. This arrangement of tiles will be called position 4.

1	2	3
4	5	6
7	8	9

Fig. 1a

4		

Fig. 1b

A move consists of sliding a tile into the empty space. From position 4, the next move will result in position 1, position 5 or position 7.

- (i) Draw a graph with nine vertices to represent the nine positions and arcs that show which positions can be reached from one another in one move. What is the least number of moves needed to get from position 1 to position 9? [3]
- (ii) State whether the graph from part (i) is Eulerian, semi-Eulerian or neither. Explain how you know which it is. [2]

[Total: 5 Marks]
[OCR A, D1, Jan 2008, Q2]



Q9.

A tetromino is a two-dimensional shape made by joining four squares edge-to-edge. Joins are along complete edges.

- (i) Represent each of the tetrominoes below by a graph in which the nodes represent the squares and two nodes are joined by an arc if the squares share a common edge. [2]



(A)



(B)



(C)



(D)

- (ii) Six simply connected graphs with four nodes are shown below. For each graph, either draw a tetromino that can be represented by the graph, as in part (i), or explain why this is not possible. [3]



(1)



(2)



(3)



(4)



(5)



(6)

Two tetrominoes are regarded as being the same if one can be rotated or reflected to form the other. Derek claims that each tetromino corresponds to a unique tree with four nodes, and each tree with four nodes corresponds to a unique tetromino. Derek's claim is wrong.

- (iii) From the diagrams above, find:

- (a) a tetromino whose graph does not correspond to a tree; [1]
 (b) two different tetrominoes whose graphs correspond to the same tree. [1]

A pentomino is a two-dimensional shape made by joining five squares edge-to-edge. Joins are along complete edges. Two pentominoes are regarded as being the same if one can be rotated or reflected to form the other. There are twelve distinct pentominoes.

- (iv) When the pentominoes are represented by graphs, as in part (i), there are only four distinct graphs. Draw these four graphs. [3]

[Total: 10 Marks]
 [OCR A, D1, Jan 2013, Q2]

Q10.

Explain why it is impossible to draw a graph with exactly four vertices of orders 1, 2, 3 and 3. [1]

[Total: 1 Mark]
 [OCR A, D1, Jan 2011, Q3i]